TIME HISTORY ANALYSIS OF DIFFERENT BRIDGE CONFIGURATIONS USING RAYLEIGH AND MODAL DAMPING

M. Abbasi and M. A. Moustafa

ABSTRACT

Energy dissipation and damping modeling of reinforced concrete (RC) bridge structures during earthquakes is crucial for nonlinear time history analysis. Rayleigh damping and Modal damping are common forms of representing linear viscous inherent damping in numerical models. This paper investigates the seismic response variations of different RC bridge configurations due to applying different damping modeling characteristics under different seismic intensities. Three damping modeling scenarios are considered in this study: Rayleigh damping with initial stiffness, Rayleigh damping with tangent stiffness, and Modal damping. The considered bridge configurations are: straight single frame bridge (S-Str), skewed single frame bridge (S-Skwd), straight multi-frame (M-Str), skewed multi-frame bridge (M-Skwd), and curved multi-frame bridge (M-Curv). The results of nonlinear response history analyses indicated that seismic responses of all the bridge configurations are affected by damping modeling characteristics. The extent of analysis results variation depends on the seismic intensity and bridge configurations.
Time History Analysis of Different Bridge Configurations using Rayleigh and Modal Damping

M. Abbasi\(^1\) and M. A. Moustafa\(^2\)

**ABSTRACT**

Energy dissipation and damping modeling of reinforced concrete (RC) bridge structures during earthquakes is crucial for nonlinear time history analysis. Rayleigh damping and Modal damping are common forms of representing linear viscous inherent damping in numerical models. This paper investigates the seismic response variations of different RC bridge configurations due to applying different damping modeling characteristics under different seismic intensities. Three damping modeling scenarios are considered in this study: Rayleigh damping with initial stiffness, Rayleigh damping with tangent stiffness, and Modal damping. The considered bridge configurations are: straight single frame bridge (S-Str), skewed single frame bridge (S-Skwd), straight multi-frame (M-Str), skewed multi-frame bridge (M-Skwd), and curved multi-frame bridge (M-Curv). The results of nonlinear response history analyses indicated that seismic responses of all the bridge configurations are affected by damping modeling characteristics. The extent of analysis results variation depends on the seismic intensity and bridge configurations.

**Introduction**

One way of defining damping in structures is considering it as mathematical approximation used to represent the energy dissipation. Damping is difficult to model accurately since it is caused by many mechanisms such as viscous effects, external friction (slippage in structural joints), internal friction (characteristic of the material type), and structural nonlinearities. Unlike the mass and stiffness characteristics of a structural system, damping does not relate to a unique physical phenomenon [1]. Typically, in engineering practice a viscous damping model is used for the sake of simplicity especially for linear elastic modal analysis.

Rayleigh damping is the most common model for viscous damping for linear and nonlinear analysis of bridges. Rayleigh damping is computationally attractive because the damping matrix preserves the sparsity pattern of the stiffness matrix and does not require formation of additional

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\(^1\)Graduate Student Researcher, University of Nevada, Reno Dept. of Civil and Environmental Engineering. MS 0258, Reno, NV 89557, Email: mabbasi@nevada.unr.edu

\(^2\)Assistant Professor, University of Nevada, Reno Dept. of Civil and Environmental Engineering. MS 0258, Reno, NV 89557, Tel: 775-682-7919; Fax: 775-784-1390; Email: mmoustafa@unr.edu

matrices. The Rayleigh damping matrix, $C$, consists of the superposition of a mass-proportional damping and a stiffness-proportional damping components. The mass-proportional component decreases the modal damping ratio with increased frequency, and the stiffness proportional term increases damping ratios with increased frequency. Initial and tangent stiffness matrices can be used in the stiffness-proportional part. Rayleigh damping modeling characteristics should be investigated and carefully selected in seismic analysis of bridges in order to avoid some consequences of inappropriate modeling such as spurious damping forces, underestimation of structure’s collapse potential, and underestimation of peak displacement demands.

In addition, Modal damping is another typical viscous damping matrix, which is suggested to be used in nonlinear response history analysis but has not been extensively investigated to replace Rayleigh damping for bridge analysis. Modal damping is constructed by superposition of modal damping matrices that leads to a fully populated damping matrix and in turn, additional computational demands are required. The open source analysis package OpenSees [2] has been recently extended to include Modal damping [3].

This study aims at investigating the implications of using different damping modeling for nonlinear time history analysis of bridges. Three damping modeling scenarios including Rayleigh damping with initial stiffness, Rayleigh damping with tangent stiffness, and Modal damping were considered. Moreover, several bridge configurations were used in this study which includes: straight single frame bridge (S-Str), skewed single frame bridge (S-Skwd), straight multi-frame (M-Str), skewed multi-frame bridge (M-Skwd), and curved multi-frame bridge (M-Curv). The different damping modeling characteristics were used to evaluate the seismic response of all the bridge configurations under three different ground motions that represent different seismic intensities: weak, moderate, and strong.

**Rayleigh Damping**

Rayleigh damping is a linear combination of the mass and stiffness matrices as follows:

$$C = a_0 m + a_1 k$$  \hspace{1cm} (1)

where $m$ and $k$ are the mass and stiffness matrices of the structure, respectively. The coefficients $a_0$ and $a_1$ are selected to specify the modal damping ratio in two modes. By assigning the same damping ratio $\zeta$ to two modes with frequencies $\omega_i$ and $\omega_j$, the coefficients $a_0$ and $a_1$ are computed as follows:

$$a_0 = \frac{\zeta (2\omega_i \omega_j)}{(\omega_i + \omega_j)}$$  \hspace{1cm} (2)

$$a_1 = \frac{\zeta (2)}{\omega_i + \omega_j}$$  \hspace{1cm} (3)

Given $a_0$ and $a_1$, the damping ratio in any vibration mode with frequency $\omega_n$ can be computed by:

$$\zeta_n = \frac{(a_0)/(2\omega_n) + (a_1/\omega_n)}{(2)}$$  \hspace{1cm} (4)

The damping ratio is constrained to be less than $\zeta$ for the frequency band between $\omega_i$ and $\omega_j$, but outside of this frequency band, particularly for frequencies less than $\omega_i$, damping ratios can
significantly vary outside the specified frequency band as shown in Figure 1.

\[ \zeta_n = \frac{a_0}{2\omega_n} + \frac{a_1\omega_n}{2} \]  

Figure 1. Variation of modal damping ratios with natural frequency in Rayleigh damping

**Bridge Specifications and Model Details**

This study considered three-span straight single frame bridge, three-span skewed single frame bridge, four-span skewed multi-frame bridge, straight multi-frame and four-span curved multi-frame bridge with seat type abutments due to their prevalence in California. In the selected bridges, the columns and deck are monolithic, while the deck at the seat type abutments rests on elastomeric bearing pads. Expansion joints exist between the deck and abutments. In each bent, columns are supported on pile caps with a group of piles underneath it. External shear keys are integral parts of the bridge to transfer the shear forces between the superstructure and substructure. According to typical Caltrans design and modeling practice [4], a pin connection between the two-column bent and foundation pile cap is considered. Three dimensional analytical finite element models were developed for different bridge configurations using OpenSees [2], which involved both geometrical and material nonlinearities, to evaluate the seismic behavior of the bridges considering different damping modeling and damping ratios. Figure 2 schematically summarizes the five different considered configurations, with the skewed and curved bridges viewed in plan. In addition, Figure 3 shows a general layout of the bridge configuration and the Opensees analytical model characteristics used in this study.

Figure 2. Different bridge configurations considered in this study: (a) straight single frame (S-Str), (b) skewed single frame (S-Skwd), (c) skewed multi-frame (M-Skwd), (d) straight multi-frame (M-Str), and (e) curved multi-frame (M-Curv)
The deck elements were modeled using elastic beam column elements since the superstructure is required to remain essentially elastic during seismic events. Effective width of the superstructure was considered to calculate cross-section properties and assigned to the longitudinal deck elements. Elastomeric bearing pads are the most commonly used bearing types in this class of bridges, which typically transfer horizontal forces to the substructure by friction. Therefore, their response can be characterized by elastic perfectly plastic material [5, 6]. In addition, in order to model the potential pounding which is a prevalent phenomenon at the seat type abutment, impact elements were considered [7]. A bilinear model proposed by Muthukumar [8] was used to model the impact between the deck and the abutment back-wall. The model chosen for the shear keys was based on the work done by Megally et al. [9]. Nonlinear beam column elements with fiber-defined cross sections were assigned to the columns. Reinforcement steel was modeled using the “Reinforcing Steel” material, which was developed from the base model proposed by Chang and Mander [10]. The behavior of reinforced concrete was modeled using Concrete 07, which is again based on the model proposed by Chang and Mander [10]. The translation and rotational springs were modeled using simple linear springs and were assigned to zero length elements at the base of the columns. The pinned connection detail of the column base for multi-column bent bridges was considered. Rigid links were used to connect the columns top
to the deck elements due to the superstructure and the substructure integrity. A hyperbolic gap material which is based on the study of Shamsabadi and Yan [11] was considered for soil passive resistance of the abutments. The abutment piles were modeled by tri-linear springs based on recommendation of Choi [12].

Results and Discussions

To provide insight into the effect of damping characteristics on the bridge seismic evaluation, nonlinear time history analysis (NLTHA) of the bridge models was performed in Opensees. The ground motion time histories of the three pairs used in this analysis in the fault-normal and fault-parallel directions are shown in Figure 4.

Figure 4. Fault-parallel (left) and fault-normal (right) components of the selected ground motions for three intensity levels: (a) weak; (b) moderate; and (c) strong earthquakes

Figure 5 shows the displacement response history of the deck in the transverse direction for the three-span straight single-frame bridge by using three damping modeling characteristics under three earthquake levels. According to Figure 5, changing the damping modeling from Rayleigh damping with initial stiffness (RDI) to Modal damping caused the reduction of the deck displacement of the straight single-frame bridge by 40%, 23%, and 3% under the weak, moderate and strong seismic loads, respectively. The moment–curvature response for one of the intermediate
pier columns in the transverse direction under all the earthquake levels for straight single-frame bridge cases is compared in Figure 6. According to Figure 6, the maximum curvature demands were reduced by about ~30% when the damping modeling characteristic changed from RDI to Modal damping.

Figure 5. Deck displacement history of straight single frame bridge with different damping modelling under: (a) weak; (b) moderate; and (c) strong earthquakes in transverse direction

Figure 6. Column moment-curvature response in the Transverse direction for three-span straight under: (a) weak; (b) moderate; and (c) strong earthquakes

For a comprehensive and better visual comparison, Figure 7 shows the peak column
curvature and top pier displacement in the transverse direction for three different bridge configurations that include the straight (S-Str), skewed single-frame (S-Skwd) and straight multi-frame (M-Str) bridge for different damping modeling characteristics and earthquake intensities.

Figure 7. Peak response in transverse direction, peak column curvature (left) and top-pier displacement (right) under (a) weak; (b) moderate; and (c) strong earthquakes for the straight (S-Str), skewed single-frame (S-Skwd) and straight multi-frame bridge (M-Str), respectively.

The above figure shows that the peak displacement and curvature under weak and moderate
earthquakes is more sensitive to changing the damping modeling rather than strong seismic load. According to Figures 7a and 7b, the seismic response of three-span straight single-frame (S-Str) is the most sensitive bridge configuration to changing the damping modeling characteristic under the weak and moderate seismic loads. In addition, Rayleigh damping with tangent stiffness (RDT) and the RDI shows a negligible sensitivity to changing damping modeling characteristics. However, according to Figure 7c, a change of bridges seismic demands can be expected by changing RDI to RDT for all the bridge configurations.

Figure 8 illustrates a comparison of peak base-shear in longitudinal and transverse directions of different bridge configurations across all the damping modeling characteristics under different seismic loads. According to Figure 8, the effect of damping modeling characteristics on the seismic demands of the bridges is a function of both the seismic load intensity and bridge configuration. For instance, changing Modal to RDT damping caused 124% reduction of peak base-shear value in longitudinal direction for the four-span multi-frame curved bridge (M-Curv) under weak earthquake. However, a much smaller change (~12% reduction) can be seen for this bridge configuration under the strong intensity earthquake.

**Conclusions**

In this paper, the effect of three damping procedures on the seismic response of different bridge configurations under different seismic loads was investigated. The three considered damping modeling scenarios were Rayleigh damping with initial stiffness, Rayleigh damping with tangent stiffness, and Modal damping. The considered bridge configurations were: straight single frame bridge, skewed single frame bridge, skewed multi-frame bridge, straight multi-frame and curved multi-frame bridge. The results showed that the seismic response of all the bridge configurations were affected by damping modeling characteristics. However, the extent of sensitivity is a function of the seismic intensity and bridge configurations. Bridge designers can use these preliminary results to better understand the consequence of each damping modeling on seismic response evaluation of various bridge configuration.
Figure 8. Peak base-shear in longitudinal (left) and transverse (right) direction, under (a) weak; (b) moderate; and (c) strong earthquakes for the straight (S-Str), skewed single-frame (S-Skwd) and Skewed (M-Skwd) and curved multi-frame (M-Curv) bridge, respectively.

References

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