PROBABILISTIC BLIND IDENTIFICATION OF SOIL-STRUCTURE SYSTEMS USING EXTENDED KALMAN FILTER

A. Jabini\textsuperscript{1}, M. Mahsuli\textsuperscript{2} and S.F. Ghahari\textsuperscript{3}

ABSTRACT

This paper puts forward a probabilistic approach for simultaneous identification of the dynamic stiffnesses of the soil-foundation and the foundation input motion of soil-structure systems from sparsely-measured structural responses. The soil-structure system in this approach is represented by a shear building resting on sway- and rocking-representative springs and dashpots. The proposed approach employs extended Kalman filtering in time domain. This method accounts for two major sources of uncertainty in the problem: the system noise and the measurement noise. In the proposed method, the state vector of the system is augmented by unknown parameters of the soil-structure system, including the soil-foundation representative springs and dashpots. In each time step, a threefold procedure is carried out. First, the current states of the system are predicted by solving the equation of motion in which the posterior estimates of the dynamic stiffnesses and the input of the previous time step are employed. Second, the input of the current time step is estimated using an unbiased, minimum-variance estimator. Third, the current states and the dynamic stiffnesses of the system are updated to obtain a posterior estimate of the dynamic stiffness at the current time step. This iterative process of prediction and updating is repeated for the full duration of the excitation. The end results are the mean vector and the covariance matrix of the unknowns, including the soil-structure parameters and the foundation input motion. Such results have direct applications in risk and reliability analysis of flexible-base structures and rapid damage detection of a building portfolio following an earthquake.

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Probabilistic Blind Identification of Soil-Structure Systems Using Extended Kalman Filter

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This paper puts forward a probabilistic approach for simultaneous identification of the dynamic stiffnesses of the soil-foundation and the foundation input motion of soil-structure systems from sparsely-measured structural responses. The soil-structure system in this approach is represented by a shear building resting on sway- and rocking-representative springs and dashpots. The proposed approach employs extended Kalman filtering in time domain. This method accounts for two major sources of uncertainty in the problem: the system noise and the measurement noise. In the proposed method, the state vector of the system is augmented by unknown parameters of the soil-structure system, including the soil-foundation representative springs and dashpots. In each time step, a threefold procedure is carried out. First, the current states of the system are predicted by solving the equation of motion in which the posterior estimates of the dynamic stiffnesses and the input of the previous time step are employed. Second, the input of the current time step is estimated using an unbiased, minimum-variance estimator. Third, the current states and the dynamic stiffnesses of the system are updated to obtain a posterior estimate of the dynamic stiffness at the current time step. This iterative process of prediction and updating is repeated for the full duration of the excitation. The end results are the mean vector and the covariance matrix of the unknowns, including the soil-structure parameters and the foundation input motion. Such results have direct applications in risk and reliability analysis of flexible-base structures and rapid damage detection of a building portfolio following an earthquake.

Introduction

This study proposes a probabilistic approach for identification of the dynamic stiffnesses of the soil-foundation system and the foundation input motion using sparse measurements of structural responses. It is well established in the literature that the flexibility of soil underneath structures notably affects the structural performance [1]–[5]. This emphasizes the need for proper modeling of the effect of soil-structure interaction (SSI) when evaluating the seismic performance of structures. In turn, proper modeling of SSI requires accurate estimations of the dynamic stiffnesses of the soil-foundation system.

In recent decades, numerical/analytical [6]–[8] and experimental methods [9] have been proposed to assess the dynamic stiffnesses of the soil-foundation. The numerical and analytical methods are usually limited to simple cases and do not incorporate real-life data. Furthermore,

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scale effects overshadow the results obtained by experimental methods.

Few methods are able to incorporate real-life data. Ghahari et al. [10], [11] proposed a deterministic modal identification approach to estimate the soil-foundation impedance function using measurements of structural response. In a more recent study, Shirzad et al. [12] devised a Bayesian approach to identify dynamic stiffnesses of the soil-foundation based on model updating in frequency domain while accounting for the prevailing uncertainties. The issue with modal identification methods is that they require an additional step of identifying the mode shapes and frequencies of the system prior to the modal identification.

Identification methods in time-domain have distinct advantages over the modal methods. They directly identify unknown parameters and can be applied to nonlinear structures. Amongst the identification methods in time domain, stochastic filtering, and in particular, the Kalman filtering family of methods are able to account for the major sources of uncertainty in system identification. The Kalman filter is derived from Bayesian updating. It comprises two consecutive stages of prediction and updating the state of dynamic systems at each time step in the presence of two sources of uncertainty: system noise, which represents the uncertainty in the prediction model, and the measurement noise, which accounts for the uncertainty in measured responses. At each time step, a prior estimate of the states of the system is predicted through the prediction equation, e.g., equations of motion in state space, using the posterior estimate of states of the system at the previous time step. Thereafter, using the measurements of some of the states at the current time step, the predicted states are updated based on the Bayes’ theorem. In fact, the difference between the predicted states and the measured states of the system is used to update the prior estimate of the state to a posterior value. For this purpose, a gain matrix is used that picks an optimal point between the prediction and the measurement. This gain matrix is determined by minimizing the sum of the variances of the states. Hence, a Kalman filter is a minimum-variance filter.

Expanding the fundamental work of Kalman and Bucy [13], [14] in linear filtering theory, Jazwinski [15] proposed the extended Kalman filter for parameter estimation by augmenting the vector of system states with unknown parameters. Hoshyia and Saito [16] employed the extended Kalman filter with weighted global iteration for parameter estimation of linear and inelastic bilinear multi-degree-of-freedom (MDOF) systems. These methods require advance knowledge of the input excitations. However, measurements of site-specific excitations induced by the likes of earthquake and wind are usually unavailable. To tackle this issue, Wang and Haldar [17] were the first to propose an output-only system identification method for linear systems using the extended Kalman filter. In their approach, input was absent in the observation equation, i.e., there was no “direct feedthrough” of the input in the observation equation. Gillijns and De Moor [18], [19] proposed a minimum-variance method for simultaneous identification of the state and unknown input for cases with and without direct feedthrough in the observation equation. In these methods, rank-deficiency of the feedthrough matrix led to convergence issues for systems with direct feedthrough in the observation equation. Lei et al. [20]–[22] and Pan et al. [23], [24] proposed general Kalman filters to address the issue of dealing with rank-deficient feedthrough matrices.
In the present study, the extended Kalman filter with unknown input and without direct feedthrough, dubbed the EKF–UI–WDF [24], is employed for simultaneous estimation of the soil-foundation impedance functions and the foundation input motion. This approach results in the mean estimates of the impedance functions and the foundation input motion as well as the associated covariance matrix as a measure of uncertainty in the identified values. Tracking the changes in the identified stiffness parameters of the soil-structure system can be used to monitor the health of the structure over time.

### Methodology

#### Problem Formulation

Consider an \( m \)-degree-of-freedom (DOF) structure, which is under unknown excitation. The equation of motion is expressed as

\[
\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = \eta^* f^*(t) + \eta f(t)
\]

(1)

where \( f^*(t) = [f_1^*(t) \ldots f_r^*(t)]^T \) is the vector of \( r \) unknown excitations, \( \eta^* \) is an \( m \times r \) mapping vector, \( f(t) = [f_1(t) \ldots f_s(t)]^T \) is the vector of \( s \) known excitations, and \( \eta \) is a \( m \times s \) mapping vector. In order to express the model equation in state space, consider an extended state vector of dimension \( 2m + n \)

\[
\mathbf{z} = [\mathbf{x} \ \dot{\mathbf{x}} \ \mathbf{\theta}]^T
\]

(2)

in which \( \mathbf{\theta} = [\theta_1 \ldots \theta_n] \) is the vector of unknown parameters. Using the introduced state vector above, one can rewrite Eq. 1 in state space as

\[
\dot{\mathbf{z}} = g(\mathbf{z}, f^*(t), f(t)) + \mathbf{w}(t)
\]

(3)

in which the nonlinear state function for this problem becomes

\[
g(\mathbf{z}, f^*(t), f(t)) = \begin{bmatrix} \dot{x} \\ \mathbf{M}^{-1} \left(-\mathbf{Kx} - \mathbf{Cx} + \eta^* f^*(t) + \eta f(t)\right) \\ 0_{n \times 1} \end{bmatrix}
\]

(4)

\( \mathbf{w}(t) \) is zero-mean Gaussian system noise with covariance matrix \( \mathbf{Q}_k \). Also, Discretized observation equation is expressed as:

\[
y_{k+1} = h(\mathbf{z}_{k+1}, f^*_{k+1}, f_{k+1}) + \mathbf{v}_{k+1}
\]

(5)

where \( y_{k+1} \) is \( l \)-observation output vector at \( t = (k+1)\Delta t \) in which \( \Delta t \) is sampling time interval. \( \mathbf{v}_{k+1} \) is zero-mean Gaussian measurement noise vector with covariance matrix \( \mathbf{R}_k \).
Model

The representative model used for soil-foundation-structure, illustrated in Fig. 1(a), is a shear building on sway and rocking springs and dashpots as shown in Fig. 1(b). The only input considered here for this system is the sway component of the foundation input motion. In this system, there are \( n_f \) DOFs representing soil-foundation system and \( n_s \) DOFs representing superstructure. The equation of motion in the relative coordinate system is expressed as

\[
\begin{bmatrix}
    m_f & m_{f-s} \\
    m_{s-f} & m_s
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_f \\
    \ddot{u}_s
\end{bmatrix}
+ \begin{bmatrix}
    c_f & 0 \\
    0 & c_s
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_f \\
    \dot{u}_s
\end{bmatrix}
+ \begin{bmatrix}
    k_f & 0 \\
    0 & k_s
\end{bmatrix}
\begin{bmatrix}
    u_f \\
    u_s
\end{bmatrix}
= \begin{bmatrix}
    m_f & m_{f-s} \\
    m_{s-f} & m_s
\end{bmatrix}
\mathbf{s}\mathbf{u}_f
\]

in which sub-matrices \( m_f, m_{f-s}, m_s, c_f, c_s, k_f, k_s \) are calculated according to [25] and \( s = [-1 \ 0 \ 1_{1 \times n_s}]^T \) is the input influence vector. The subscript \( f \) refers to the foundation degrees of freedom, i.e., sway and rocking, and the subscript \( s \) refers to the superstructure.

Assuming the mass matrix to be known and the damping matrix of the superstructure to be proportional to the stiffness matrix with the coefficient of \( a_1 \), the unknown parameter vector is selected as \( \mathbf{\theta} = [k_h \ k_t \ k_1 \ \cdots \ c_h \ c_t \ a_1]^T \). Augmenting the state vector with unknown parameters, Eq. 6 is transformed to the state space according to Eq. 4, as follows:

Figure 1. (a) Soil-structure system and (b) representative shear building model on sway-rocking springs and dashpots
\[
g(Z, f^*(t), f(t)) = \begin{bmatrix} \dot{x} \\ M^{-1}(-Kx - C\dot{x}) + sf^*(t) \\ 0_{n\times1} \end{bmatrix}
\]  

(7)

Since the absolute acceleration is commonly measured in instrumented structures, it is considered as the observation and Eq. 5 is written as

\[
y_{k+1} = M^{-1}(-Kx - C\dot{x})
\]

(8)

The objective here is to identify \(f^*(t)\) and \(\theta\). To this end, the EKF-UI-WDF algorithm, which is summarized in Table 1, is employed.

<table>
<thead>
<tr>
<th>From (k = 1) to (t_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Update</strong></td>
</tr>
<tr>
<td>(1) (\hat{X}_{k+1</td>
</tr>
<tr>
<td>(2) (F_{k</td>
</tr>
<tr>
<td>(3) (P_{k+1</td>
</tr>
<tr>
<td><strong>Measurement Update</strong></td>
</tr>
<tr>
<td>(4) (K_{Z,k+1} = P_{k+1</td>
</tr>
<tr>
<td>(5) (S_{k</td>
</tr>
<tr>
<td>(6) (\hat{f}^*_{k+1</td>
</tr>
<tr>
<td>(7) (\hat{Z}_{k+1</td>
</tr>
<tr>
<td>(8) (P_{Z,k+1</td>
</tr>
</tbody>
</table>

**Validation**

To validate the proposed approach, a synthetic example is employed in which a 5-story shear building rests on sway- and rocking-representative springs and dashpots. This soil-structure system is subjected to the Nahanni 1985 [9] ground motion recorded at the NWT station No. 3 with a sampling frequency of 200 Hz, PGA of 0.148g, and a predominant period of 0.062 sec. This record is employed here as the sway components of the foundation input motion. The acceleration time history of this record is shown in Fig. 2 with blue curve. The red curve will be addressed shortly.
The resulting absolute accelerations at the DOFs of sway and the 1st, 3rd and 5th stories are then computed and polluted with noise to be used as the observation channels in this synthetic example. A white noise with signal to noise ratio (SNR) of 30 is used for this purpose. These observation channels are then employed to identify the dynamic stiffnesses of the soil-structure system and the input excitation, which are here regarded as unknowns.

The parameters and states are scaled to avoid ill-conditioned matrices. The initial trial values for the system parameters are assumed to have an error of 30% compared to their true values. The covariance matrix of the measurement noise and the system noise are set to be

\[
R = 3 \times 10^{-5} I_4 \text{ m/s}^2 \quad \text{and} \quad Q = \begin{bmatrix} 0_{14 \times 14} & 0_{14 \times 10} \\ 0_{10 \times 14} & 10^{-12} I_{10 \times 10} \end{bmatrix},
\]

respectively.

The estimated unknown input, which is plotted with red curve, is compared with the true input in time and frequency domains in Fig. 2. As seen, the estimated input is in reasonable agreement with the true input. For instance, the root-mean-square of errors in input estimation is 0.7% of norm of the input, which is satisfactory. The variations of the estimated parameters over time only for \([k_1 \; k_2 \; k_3 \; k_4 \; k_5 \; c_1 \; a_1]\) are plotted in Fig. 3 for brevity. In this figure, the blue lines represent the true values, the red curves represent the mean estimate, i.e., the most likely realization of the estimate, and the dashed lines represent the mean ± one standard deviation estimates and are presented to reveal the estimation uncertainty. Note that these plots are based on normalized values. The corresponding errors in the mean estimates are tabulated in Table 2.

![Figure 2. Estimated unknown input in (a) time domain and (b) frequency domain](image)
Figure 3. Estimated normalized parameters
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k_h$</th>
<th>$k_r$</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$k_5$</th>
<th>$c_h$</th>
<th>$c_r$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Error (%)</td>
<td>30.00</td>
<td>-30.00</td>
<td>-30.00</td>
<td>30.00</td>
<td>30.00</td>
<td>30.00</td>
<td>-30.00</td>
<td>-30.00</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>Final Error (%)</td>
<td>-0.02</td>
<td>-0.06</td>
<td>-0.68</td>
<td>1.32</td>
<td>-0.26</td>
<td>0.02</td>
<td>-0.13</td>
<td>-0.96</td>
<td>30.00</td>
<td>1.51</td>
</tr>
</tbody>
</table>

As can be seen in Table 2 all parameters except the damping coefficient in the rocking degree of freedom are identified reasonably accurately. Further investigation showed that the value of $c_r$ has almost no contribution to measured responses. For further investigation, the level of identifiability of the parameters are assessed to assess their influence on the response of the structure. To this end, the sensitivity of measurements to each parameter is computed and presented in Fig. 4. As seen, the results of sensitivity-based identifiability analysis suggest that the absolute acceleration at all degrees of freedom are nearly insensitive to $c_r$. In other words, $c_r$ has no contribution to measured responses and therefore is not identifiable. Table 3 presents the resulting modal frequency and damping ratios computed from the estimation is Table 2. As seen, all modal parameters are identified with acceptable accuracy.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final error in modal frequency (%)</td>
<td>0.00</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.04</td>
<td>0.13</td>
<td>0.06</td>
<td>-0.01</td>
</tr>
<tr>
<td>Final error in modal damping (%)</td>
<td>-0.84</td>
<td>-0.95</td>
<td>-0.50</td>
<td>1.30</td>
<td>0.77</td>
<td>1.68</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 4. Total information for each parameter in observations
Conclusion

A probabilistic approach for simultaneous identification of soil-foundation dynamic stiffnesses and the foundation input motion of soil-structure systems is proposed. In this approach, a shear building supported by sway-rocking springs and dashpots models the soil-structure system. This simplified model significantly reduces the number of DOFs used in modeling the soil-structure system compared to such complex models as finite elements, while yielding reasonably accurate results. This approach yields the probability distributions of the unknowns. Such results are immediately applicable to risk and reliability analysis of structures as well as probabilistic damage detection. Since the proposed approach relies on real-life observations, it candidly incorporates the uncertainties of the soil-structure system. Estimation of the foundation input motion is another important result of the proposed approach. Furthermore, a sensitivity-based identifiability analysis is presented to investigate the degree of contribution of different unknown parameters to the responses of the system, which reveals how well they can be identified using the proposed approach. As a limitation, the presented methodology requires tuning the filter, i.e., setting proper values for the covariance matrices of the process and measurement noises. Future research should address the identification of frequency-dependent dynamic stiffness of the soil-foundation system.

Acknowledgements

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