HYSTERESIS MODEL OF RHS COLUMNS INCLUDING DETERIORATING BEHAVIOR GOVERNED BY LOCAL BUCKLING

S. Yamada¹, T. Ishida² and Y. Jiao³

ABSTRACT

A hysteretic model of rectangular hollow section (RHS) columns that includes the deteriorating range caused by local buckling is proposed. The proposed model consists of the skeleton curve, the Bauschinger part that appears before reaching the maximum strength, the strength increasing part of the deteriorating range, and the unloading part. Of these, the skeleton curve, including the deterioration range caused by local buckling, which is considered to be equivalent to the load-deformation relationship under monotonic loading, is obtained through an analytical method. Bilinear hysteretic models based on experimental results are applied to the Bauschinger part and the strength increasing part. The elastic stiffness is applied to the unloading part. The proposed model is verified by comparing with experimental results of RHS columns under monotonic and cyclic loading.

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Introduction

Earthquake response analysis is a useful tool for evaluating the seismic performance of buildings. As the accuracy of the hysteretic model applied to the analysis is improved, the reliability of the analysis result also improves. The most fundamental and reliable method for evaluating the behavior of steel members is the use of modeling based on experimental results. The range of application of the hysteretic model based on experimental results is determined by the range of parameters, for example, the width-to-thickness ratio, axial force and the loading protocol. Even with the combination of different width-to-thickness ratios and axial force only, a large number of experimental results are necessary. In addition, the loading protocol is an important factor. Generally, cyclic loading tests of steel members have been conducted with the standard type loading protocol, for example, an incremental amplitude protocol or a constant amplitude protocol.

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To evaluate the performance of the members, employing the standard type loading protocol is effective for comparing the influence of different parameters; however, it might not be able to reproduce the random loading protocol (e.g., the seismic behavior of structures). Constructing a general hysteretic model of steel members that reflects many parameters is an important but difficult problem to solve.

In this study, the behavior of RHS columns under random loading protocol is evaluated. The hysteretic model examined in this study consists of the skeleton curve, which corresponds with the load-deformation relationship under monotonic loading, the Bauschinger part, the strength increasing part in the deteriorating range and the unloading part. The skeleton curve includes the deteriorating range obtained through numerical analysis based on the behavior of stub-columns under monotonic loading while considering the effects of material properties affected by the manufacturing process of RHS members, the geometrical condition (including the width-to-thickness ratio), and the applied axial force level. The bi-linear hysteretic model based on experimental results of cyclic loading tests of RHS columns under various loading histories are used to simulate the Bauschinger part before the maximum strength and the strength increasing part in the deteriorating range. The proposed hysteretic model is verified by comparison with the experimental results of RHS columns under various loading histories.

**Hysteretic Rule of RHS Columns under Random Cyclic Loading**

Hysteresis loops of steel members under random cyclic loading can be decomposed into the skeleton curve, the Bauschinger part (in the deteriorating range, it is defined as the strength increasing part), and the unloading part, as shown in Fig. 1 [1], [2]. Before the member reaches its maximum strength, the skeleton curve is obtained by connecting parts of the load-deformation relationship of both positive and negative sides sequentially when the member experiences its highest loading for the first time. After the member reaches the maximum strength, the skeleton curve is obtained as the envelope curve of the cumulative load-deformation relationship on both positive and negative sides.

In the case of RHS columns, (1) the skeleton curve including the deteriorating range governed by local buckling is predicted through analysis as the load-deformation relationship under monotonic loading. The analytical procedure is expressed in the following part. (2) For the Bauschinger part before maximum strength, a bi-linear model proposed by Akiyama and Takahashi [2] is applied. For the strength increasing part in the deteriorating range, a bi-linear model based on experimental results [3] is introduced. Both models are introduced in the following part. (3) Elastic stiffness is applied in the unloading part.

**Load-Deformation Relationships of RHS Members under Monotonic Loading Considering Deteriorating Behavior caused by Local Buckling through Analyses**

**Outline of Analytical Method**

The load deformation relationships of RHS members under monotonic loading, including the deteriorating range governed by local buckling, are predicted by in-plane analysis of the ideal cantilever member under the following assumptions:

1) The assumption of a plane section.
2) The deformation caused by the shear force remains elastic.
3) There is no out-of-plane deformation of the member.
Fig. 1 Decomposition of the hysteresis of steel members

The analytical method is summarized as follows:

1) The moment-curvature relationship can be obtained by integrating the stress over the cross section. To determine the stress-strain state of the cross section, the stress-strain curve of the stub-column is applied to the compressive part of the cross section, and the stress-strain curve of the material is applied to the tensile part of the cross section.

2) The analytical model, shown in Fig. 2, is divided into small elements along the whole length, and it is assumed that the curvature is constant in one element. The load-deformation relationship is obtained by numerical integration of the curvature using the following equations:

\[ \theta_{i+1} = \theta_i + \phi_i \cdot \Delta x \]  
\[ y_{i+1} = y_i + \theta_i \cdot \Delta x + \frac{1}{2} \cdot \phi_i \cdot \Delta x^2 \]  
\[ M_{i+1} = M_i - P \cdot (\theta_i \cdot \Delta x + \frac{1}{2} \cdot \phi_i \cdot \Delta x^2) - Q \cdot \Delta x \]

where \( \phi_i \): curvature of the element, \( \theta_i \): rotation of the element, \( y_i \): displacement of the element, \( \Delta x \): length of the element, \( M \): moment of the element, \( P \): axial force, \( Q \): shear force.

3) The point at which the stress obtained by the stub-column test is assumed to be located at the position of a quarter wavelength apart from the starting point of the local buckling is shown in Fig. 3. On the condition that the stress produced by axial force and shear force reaches the maximum stress of the equivalent stub-column, the maximum strength of the member is determined. The cross section of the equivalent stub-column is assumed to be as follows:

\[ \left( \frac{D}{t} \right)_e = \sqrt{\frac{1 + k^2}{2}} \cdot \left( \frac{D}{t} \right) \]

where \( k \): ratio of the area of the compressive part of the web plate to the total area of the web plate under full plastic moment.

4) In the deteriorating range, the model is divided into two parts, i.e., local buckling domain and elastic unloading domain, as shown in Fig. 3. In the former, plastic deformation progresses, whereas in the latter, elastic unloading occurs. The length of the local buckling domain \( L_b \) corresponds to the length of the plastic hinge region in the deteriorated range and governs the behavior of the member. In this analysis, it is assumed that the length of the local buckling domain
$L_b$ is the same as half of the local buckling wavelength generated in the RHS member subject to bending and compression.

5) The stiffness of the stress-strain curve in the deteriorating range is controlled as follows:
   a) In the compressive part of the local buckling domain, in the case of compressive strain increasing, a deteriorating stiffness of the equivalent stub-column is given. In the case of compressive strain decreasing, the elastic stiffness is given.
   b) In the tensile part of the local buckling domain, in the case of tensile strain increasing, the stiffness of the material is given. In the case of tensile strain decreasing, the elastic stiffness is given.
   c) In the elastic unloading domain, the elastic stiffness is given.

Stress-strain relationships of RHS members under compressive stress

Stress-strain relationships of RHS members under compressive stress considering local buckling were modeled based on stub-columns test results [4]. The test results involve 35 cold roll-formed RHS columns. Two steel grades of minimum specified tensile strength of 400 N/mm$^2$ and 490 N/mm$^2$ were used in the test. The length of the specimen $L_s$ is 3 times the depth of cross-section $D$. All specimens were fabricated without annealing.

The axial stress of the specimen was calculated by dividing the axial load by the area of the original section of the specimen. The axial strain of the specimen before the maximum strength was calculated by dividing the axial deformation of the specimen by the original length $L_s$. In the post-buckling deteriorating range, axial deformation of the stub-column progresses only at the part of local buckling generated, and other parts are unloaded elastically. Therefore, the increment of axial strain was calculated by dividing the increment of axial deformation of the specimen by the length of the buckled part $L_{sb}$. (Fig. 4)

A typical stress-strain curve of a stub-column is shown in Fig. 4, where $\sigma_{yc}$: the yield stress of the stub-column, $\sigma_{uc}$ ($= S \cdot \sigma_{yc}$): the maximum strength of the stub-column, $\sigma_{tc}$ ($= T \cdot \sigma_{yc}$):
the stress of the stiffness transition point in the deteriorating range, \( E \): Young’s modulus, \( \varepsilon_{yc} \): the elastic strain corresponding to the yield stress of the stub-column, \( \varepsilon_{uc} \): the strain corresponding to the maximum strength of the stub-column, \( \mu \) (\( = \varepsilon_{uc}/\varepsilon_{yc} \)): the strain ductility factor, \( E_{d1} \): the negative stiffness of the first part of the deteriorating range, \( E_{d2} \): the negative stiffness of the second part of the deteriorating range.

The stress-strain curve up to the maximum strength point can be represented by that of the stub-column with a small width-to-thickness ratio. Next, on the basis of the test results, the deteriorating behaviors of stub-columns were expressed by the following simple formulas, which were obtained by regression analysis of the test results.

\[
\mu_0 = 8.7 / \alpha - 1.2 \quad (2.62 \geq 1 / \alpha \geq 0.19) \quad (5)
\]
\[
E_{d1} / E = -0.014 \cdot \alpha^2 - 0.005 \quad (3.14 \geq \alpha \geq 0.33) \quad (6)
\]
\[
E_{d2} / E = -0.005 \quad (3.70 \geq \alpha \geq 0.62) \quad (7)
\]
\[
T / S = -0.079 \cdot \alpha + 0.81 \quad (3.70 \geq \alpha \geq 0.62) \quad (8)
\]

where \( \alpha \) (\( = \varepsilon_{y} \cdot (D / t)^2 \)): the standardized width-to-thickness ratio of the RHS members.

Furthermore, the negative stiffness of the third part of the deteriorating range is modeled based on the hysteretic behavior of the columns in the large deformation region [3].

\[
E_{d3} / E = -0.001 \quad (9)
\]
\[
T_2 / T = 0.9 \quad (10)
\]

**Wavelength of the local buckling of RHS members**

In this analysis, the region where local buckling occurs is defined as the local buckling domain, where deformation progresses in the deteriorating range, and the position of a quarter wavelength from the starting point of local buckling is set as the point where maximum strength is determined. The length \( L_b \) of the local buckling domain that corresponds to the half wavelength of local buckling generated in the RHS columns is the important parameter in this analysis. The half wavelength of local buckling generated in the roll-formed RHS columns subjected to bending and compression has been determined on the basis of experimental results to be given by the following...
Stress-strain relationships of RHS members under tensile stress

The stress-strain relationship applied to the portion under tensile stress was calculated from the stress-strain relationship of the stub-column with a small width-to-thickness ratio. Under the assumption of a constant volume of the member, the following equations can be used for conversion.

\[
\sigma_t = \sigma_y \cdot \frac{1 - \varepsilon_t}{1 + \varepsilon_t} \\
\varepsilon_t = \varepsilon_y / (1 - \varepsilon_y)
\]

where \(\sigma_y\): nominal stress (compression), \(\varepsilon_y\): nominal strain (compression), \(\sigma_t\): nominal stress (tension), \(\varepsilon_t\): nominal strain (tension)

**Hysteretic model of the Bauschinger part and the strength increasing part in the deteriorating range**

**Hysteretic model of the Bauschinger part**

The hysteretic model of the Bauschinger part by Akiyama and Takahashi [9] is shown in Fig. 5. The model is a bi-linear hysteretic model. The model is described in the portion starting from the negative side of unloading start point A. The behavior from the unloading starting point A to reaching 1/2 of the positive skeleton restoring load \(M_s^+\) (maximum load experienced on the positive side) (point B) is regarded as elastic. The skeleton restoring point C on the positive side is defined as a point where the plastic deformation from point A is \(\theta_b\) and the load is the skeleton restoring load on the positive side. \(\theta_b\) is given by Eq. (15), which is based on the experimental results.

\[
\theta_b = \frac{1 - N/N_y}{2(1 + N/N_y)} \sum \theta_s
\]

where \(N\): the axial force, \(N_y\): the axial yield strength of the section, \(\theta_s\): the cumulative plastic deformation of the skeleton curve (both the positive and negative side)

On the negative side, the unloading starting point (Point A) is the skeleton restoring point. Beyond point B, the behavior takes a second modulus oriented to point C, unless unloading starts. Next, cyclic behavior between point D and point E after unloading from between point B and point C can be described as follows. On the positive side, the behavior is elastic until the load reaches the unloading start point D. Once the load reaches point D, the behavior takes a second modulus oriented to point C, unless unloading starts. In addition, on the negative side, the behavior is elastic until the load reaches 0.5\(M_s^-\) (1/2 of the skeleton restoring load on the negative side) (Point E). Once the load reaches 0.5\(M_s^-\), the behavior takes a second modulus oriented to point A, unless unloading starts. Furthermore, cyclic behavior between point F and point G after unloading from between point E and point A can be described as follows. On the negative side, the behavior is
elastic until the load reaches the unloading start point F. Once the load reaches point F, the behavior takes a second modulus oriented to point A, unless unloading starts. On the positive side, the behavior is elastic until the load reaches \(0.5M^+_S\) (Point G). Once the load reaches \(0.5M^+_S\), the behavior takes a second modulus oriented to point C, unless unloading starts.

![Fig. 5 Hysteretic model of the Bauschinger part](image)

**Hysteretic model of the strength increasing part in the deteriorating range**

The strength increasing part in the deteriorating range was also modeled as a bi-linear curve based on experimental results [3]. In the modeling, the experimental results of not only the incremental displacement amplitude loading but also the constant displacement amplitude loading and the decremental displacement amplitude loading were used. Therefore, this model can be used to represent the behavior of RHS columns under a random loading protocol consisting of various displacement amplitudes.

Behavior from the unloading start point A on the negative side is shown in Fig. 6. From point A to point B, during which period the load reaches 60% of the positive skeleton restoring load \(M^+_1\), the behavior is elastic. The skeleton restoring point C on the positive side is defined as a point where the plastic deformation from point A is \(\theta_1\) and the load is the skeleton restoring load on the positive side. \(\theta_1\) is given by Eq. (16), based on the experimental results.

\[
\theta_1 = 0.1 \left(\frac{1-N/N_y}{1+N/N_y}\right)^2 \sum \theta_s
\]  

(16)

Alternatively, when the deformation reaches point A again from between point A and point B, the deformation returns to the skeleton curve on the negative side.

Next, in the case that the direction of deformation reverses at point D, where the plastic deformation progressed \(\Delta \theta_d\) from point B, the hysteresis is modeled as follows. The skeleton restoring point F on the negative side is a point at which the load is \(M^{-}_2\) and the deformation is the same as that of the unloading starting point A. \(M^{-}_2\) is the load at point F', where plastic deformation progresses \(\Delta \theta_d\) on the skeleton curve from the unloading starting point A. The behavior from point D to point E, where the load reaches 0.6\(M^{-}_2\), is elastic. Beyond point D, the behavior takes a second modulus oriented to point C again, unless unloading starts. On the negative side, once the load reaches 0.6\(M^{-}_2\), the behavior takes a second modulus oriented to point F, unless unloading starts.
In the case that the direction of deformation reverses at point G, where the plastic deformation progressed $\Delta \theta_d'$ from point E, the hysteresis is modeled as follows. The skeleton restoring point I on the positive side is a point at which the load is $M_1^+'$ and the deformation is the same as that of point C. $M_1^+'$ is the load at point I’, where plastic deformation progresses $\Delta \theta_d'$ on the skeleton curve from point C. The behavior from point G to point H, where the load reaches $0.6M_1^+$, is elastic. When the load is above $0.6M_1^+$, the behavior takes a second modulus oriented to point I, unless unloading starts. On the negative side, beyond point G, the behavior takes a second modulus oriented to point F again, unless unloading starts. As described above, in the deteriorated range, the progress of plastic deformation corresponds to the progress of the skeleton curve.

![Diagram](image)

**Fig. 6 Hysteretic model of the strength increasing part in the deteriorating range**

**Verification of the hysteretic behavior of RHS columns by comparison with the experimental results**

To verify the correspondence between the proposed model and the experimental results of specimens with different width-to-thickness ratios and under different axial force levels, the experimental results of cyclic loading tests of RHS columns that were reported in previous studies are used [5], [6]. A list of the specimens and information about the test parameters of the loading protocol and applied axial force are shown in Table 1. Regardless of the width-to-thickness ratio and the applied axial force, the load-deformation relationships of the RHS columns, including the cyclic behavior in the deterioration range, correspond well under both loading protocols of incremental and decremental displacement amplitude, as shown in Fig. 7.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Section</th>
<th>$D/t$</th>
<th>Loading Pattern</th>
<th>Applied Axial Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>300 × 300 × 9</td>
<td>33.3</td>
<td>Incremental Amplitude</td>
<td>0.15 $N_y$</td>
</tr>
<tr>
<td>0° dec</td>
<td></td>
<td></td>
<td>Decremental Amplitude</td>
<td>0.14 $N_y$</td>
</tr>
<tr>
<td>Col32inc</td>
<td>60 × 60 × 3.2</td>
<td>18.8</td>
<td>Incremental Amplitude</td>
<td>0</td>
</tr>
<tr>
<td>Col32dec</td>
<td></td>
<td></td>
<td>Decremental Amplitude</td>
<td></td>
</tr>
</tbody>
</table>

$N_y$: Axial Yield Strength of the Section

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Fig. 7 Comparison of the load-deformation relationships obtained with the Test Results from reference studies [5], [6]
Conclusion

This study proposed a hysteretic model for RHS columns that includes the strength deterioration caused by local buckling and validated the calibration of the proposed hysteretic model via loading tests. The proposed hysteretic model consists of the skeleton part, which corresponds with the load-deformation relationship under monotonic loading, the Bauschinger part before the maximum strength, the strength increasing part in the deteriorating range and the unloading part. The skeleton curve employs the simulated load-deformation relationship that includes the deterioration range caused by local buckling obtained by the in-plane analysis of the cantilever member. To reproduce the deterioration behavior caused by local buckling in the in-plane analysis, the wavelength of local buckling of RHS columns, which is defined as the local buckling domain, is modeled on the basis of the experimental results. The stress-strain relationship model in the deterioration range caused by local buckling based on the stub-column tests of RHS columns is applied to the in-plane analysis as the material property of the compressive part in the local buckling domain. For the Bauschinger part before the maximum strength and the strength increasing part in the deterioration range, the bi-linear model based on the investigation of the load-deformation relationships obtained by the cyclic loading test results of RHS columns under various loading protocols is employed. The unloading part before the maximum strength and in the deterioration range is regarded as elastic.

The load-deformation relationships of the proposed hysteretic model and the experimental results of RHS columns were compared to verify the effectiveness of the proposed hysteretic model. The proposed hysteretic model corresponds well to the experimental results, including the deterioration range governed by local buckling, regardless of the test parameters.

References


