UNCERTAINTY IN LARGE-SCALE MAGNETO-RHEOLOGICAL DAMPER MODELING FOR SEISMIC HAZARD MITIGATION

J. J. Liang¹, C. Chen², and Y. Xu³

ABSTRACT

Magneto-Rheological (MR) dampers have been widely studied by researchers. Rheological properties of the MR fluid change when a magnetic field is applied so as to enable smart energy dissipation. Realistic modeling of MR damper is critical for seismic hazard mitigation. Model parameters derived from experiments have inherent uncertainties due to optimization and seismic response prediction could deviate from actual structural performance due to these uncertainties. Design capacity should be based on the maximum possible output and effective capacity from the probabilistic parameters, therefore, is extremely important for computational models. In this study, four different phenomenological models for a large-scale 200 kN MR damper are evaluated and compared through probabilistically characterizing the model parameters using the Metropolis-Hasting (MH) algorithm, including the hyperbolic tangent, the Bouc-Wen, the non-parametric algebraic, and the viscous plus Dahl models. The statistics of damper model parameters are assessed through comparing with their deterministic values to evaluate the effectiveness of different models to account for uncertainty.

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Introduction

The control of structural vibrations due to earthquakes can be achieved by various means and methods of active or passive counter forces [1]. Structural control devices have been shown effective to reduce structural displacement during earthquakes. These structural control devices can be categorized as passive, active or semi-active [2]. Semi-active devices provide controllability, stability (in a bounded-input bounded-out sense), and require a low power supply, therefore, presenting an appealing option for seismic hazard mitigation. The Magneto-Rheological (MR) damper could operate in a large temperature range and produce large control forces at low velocities with a high dynamic range (the ratio between maximum force and minimum force). MR damper, therefore, presents a promising type of seismic control devices [3]. Realistic mathematical modeling is critical for the application of MR dampers for seismic hazard mitigation. Numerical modeling of MR damper can not only improve the studies of damping behavior by replacing costly and time-consuming physical testing but also help researchers develop better semi-active control laws to achieve better structural control during earthquakes. A number of mathematical models have been investigated for MR damper in the past including the Bouc-Wen [2], the Hyperbolic-Tangent [4], the viscous plus Dahl [5] and the non-parametric algebraic [6].

Computer modeling is popular for solving complex system problems in many fields such as physics, astrophysics, social science, and engineering [7]. Most of these complex systems, however, are difficult to be accurately modeled because the models are often based on unverifiable assumptions and general approximations. Moreover, the model parameters have inherent uncertainties with limited knowledge and information thus resulting in modeling errors. Uncertainty Quantification (UQ) is therefore extremely important to account for these errors and unverified assumptions and to evaluate the effects of uncertainties for the complex system.
Parameters for the existing MR damper models are generally established using optimization schemes to minimize the error between experiment and model prediction [8]. Model parameters derived from experiments, however, have inherent uncertainties due to optimization and the seismic response prediction could deviate from actual structural performance due to these uncertainties. A probabilistic characterization of model parameters that includes extreme and unlikely parameters values might provide information that could improve seismic design methodology of structures with MR dampers. Therefore, investigation and quantification of the parameter uncertainties are essential for probabilistic analysis. However, traditional experimental testing is very time consuming and expensive; so, an alternative method is proposed and used in this study to solve the complex model with very limited data and experimental result. Baxter et al. [9] explored the parameter uncertainties of the Hyperbolic-Tangent model for a large-scale 200 kN MR damper. The Metropolis [10] - Hasting [11] (MH) algorithm was used to provide a probabilistic characterization of the parameters based on experiments with current inputs of zero amps. The analysis results showed that the probabilistic characterizations will not only be useful in the design of structural control devices but could be used to compare and contrast competing models. Caicedo et al. [8] further explored the parameter uncertainties of the nonparametric algebraic model for the same MR damper using the MH algorithm. In this work MH algorithm is used to determine the parameters uncertainties for different models. A total of four experiments were used with currents of 0.5, 1.0, 1.5 and 2.5 Amps and amplitude of 1.0 inch and a frequency of 1.0 Hz. It was concluded that a single probabilistic model might not compensate for modeling errors. The parameters of four different MR damper numerical models are studied for their uncertainties when applied to emulate the same 200 kN large-scale MR damper. Four different numerical models including the hyperbolic tangent, the Bouc-Wen, the non-parametric algebraic and the viscous plus Dahl models are evaluated and compared through probabilistically characterizing of their parameters. A total of 155 experiment data sets are used to account for various frequencies, amplitudes and input currents. The results of probabilistic characterizations of models are then evaluated using a MR damper test under predefined random displacement and current input.

**Computational Models for MR Damper**

The MR damper under study is manufactured by Lord Cooperation which has 1.47 m (58 inches) in length, weights approximately 2.734 kN (615 lbs), and a stroke of 584 mm (23 inches). The damper is expected to provide control forces of over 200 kN (45 kips) controlled by various current input ranging from 0 amps to 2.5 amps [12].

**Hyperbolic Tangent Model:** The hyperbolic tangent model was originally proposed for an 8 kN electro-rheological fluid damper by Gavin et al. [4] as a simplified version of the model proposed by Gamota and Filisko [13]. More recently, Jiang et al. [12] extended the hyperbolic tangent model to the 200 kN MR damper. The hyperbolic tangent model can be described using the following equations

$$\begin{align*}
\begin{bmatrix}
\dot{x}_0 \\
\ddot{x}_0 
\end{bmatrix} &= \begin{bmatrix}
0 & 1 \\
-k_0 - k_1 & -c_0 - c_1 
\end{bmatrix} \begin{bmatrix}
x_0 \\
\dot{x}_0 
\end{bmatrix} + \begin{bmatrix}
0 & 1 \\
-k_1/m_0 & -c_1/m_0 
\end{bmatrix} \begin{bmatrix}
x \\
\dot{x} 
\end{bmatrix} + \begin{bmatrix}
0 \\
-1/m_0 
\end{bmatrix} f_0 \tanh(\dot{x}_0/V_{ref}) \\
\end{align*}$$

where $m_0$ represents the inertia of both the fluid and the moving position; $k_0$ and $k_1$ represent the post-yield and pre-yield visco-elastic stiffness, respectively; $c_0$ and $c_1$ represent the post-yield
and pre-yield visco-elastic viscous damping, respectively; \(x\) and \(\dot{x}\) represent the displacement and velocity of damper input; \(x_0\), \(\dot{x}_0\), and \(\ddot{x}_0\) represent displacement, velocity, and acceleration of damper piston end relative to the inertial mass; the Coulomb friction is a function of the velocity across the element such that \(f(\dot{x}_0) = f_0 \tanh(\dot{x}_0/V_{ref})\), where the parameter \(f_0\) is the yield force and \(V_{ref}\) is a reference velocity; and \(f\) is the output force of the MR damper. It can be observed that the hyperbolic tangent model in Eqs. (1a) and (1b) contains a total of seven independent parameters \((k_0, k_1, c_0, c_1, m_0, f_0, V_{ref})\), which vary for different current inputs.

**Bouc-Wen Model:** The Bouc-Wen model [14] [15] has been widely used by researchers due to its proven simplicity on differential equations that fits well in the complex computational analysis system [16]. The Bouc-Wen model was first introduced by Dyke et al. [3] for the MR damper and applied for the 200 kN MR damper by Jiang et al. [12] recently. The Bouc-Wen model could be described as

\[
\begin{align*}
  f &= a\dot{x} + c_0(\dot{x} - \ddot{y}) + k_0(x - y) + k_1(x - x_0) \quad (2a) \\
  \dot{f} &= c_1\dot{y} + k_1(x - x_0) \quad (2b) \\
  \ddot{z} &= -\gamma |\dot{x} - \ddot{y}| z \cdot |z|^{n-1} - \beta(\dot{x} - \ddot{y})|z|^n + A(\dot{x} - \ddot{y}) \quad (2c) \\
  \dot{y} &= [a\dot{x} + c_0\dot{x} + k_0(x - y)]/(c_0 + c_1) \quad (2d)
\end{align*}
\]

where \(k_0\) and \(c_0\) represent the stiffness and viscous damping at large velocities, respectively; \(k_1\) represents the accumulator stiffness; \(c_1\) represents damping at low velocities due to bleed or blow-by of the MR fluid between the damper piston and cylinder; \(x_0\) represents the initial displacement of spring \(k_1\); \(x\) and \(\dot{x}\) represent the displacement and velocity of damper input; \(y, \ddot{y}, z,\) and \(\ddot{z}\) are evolutionary variables in the function; and \(f\) is the output force of the MR damper. It can be observed that the Bouc-Wen model in Eqs. (2a) – (2d) contains a total of ten independent parameters \((k_0, k_1, c_0, c_1, x_0, \alpha, \beta, \gamma, n, A)\), which vary for different current inputs.

**Non-parametric Algebraic Model:** A non-parametric algebraic model was first proposed for MR damper [6] [18] [19], and more recently adopted by Jiang [20] for the 200 kN MR damper which consists of two components: is combination of a polynomial function describing maximum damping force as a function of the control current and a shape function describing the force-velocity dependence has developed. The non-parametric algebraic model could be described as

\[
\begin{align*}
  f &= \text{sign}(\dot{x})[1 - (e^{-\alpha|\dot{x}|})] \cdot (m|\dot{x}| + b) \quad (3)
\end{align*}
\]

where \(m\) is the slope and \(b\) is the y-intercept parameter; \(1 - (e^{-\alpha|\dot{x}|})\) represents the shape description function of force-velocity dependence; \(\alpha\) simulates the correlation of hysteretic force-velocity relationship; and \(f\) is the output force of the MR damper. It can be observed that non-parametric algebraic model in Eq. (3) contains a total of three parameters \((m, b, \alpha)\), which vary for different current inputs.

**Viscous Plus Dahl Model:** The Dahl model was proposed by Dahl [5] and by Bouc [14] independently for Coulomb frictional behavior and hysteresis phenomena. The viscous plus Dahl model was first proposed by Aguirre et al. [21] for small-scale MR dampers and then by Jiang et al. [12] for the 200 kN MR damper using the following equations

\[
\begin{align*}
  \dot{w} &= \rho(\dot{x} - |\dot{x}|w) \quad (4a) \\
  f &= \kappa_x \dot{x} + \kappa_w w \quad (4b)
\end{align*}
\]
where \( w \) and \( \dot{w} \) represent the nonlinear behavior of the damper; \( \dot{x} \) represents the damper piston velocity; \( \kappa_x \) represents the viscous friction coefficient; \( \kappa_w \) represents the dry friction coefficient; \( \rho \) represents current independent parameter; \( f \) is the output force of the MR damper. It can be observed that the viscous plus Dahl model in Eqs. (4a) and (4b) contains a total of three parameters \((\kappa_w, \kappa_x, \rho)\), which vary for different current inputs.

**Probabilistic Analysis Methodology**

Bayes’ Theorem describes the probability of an event based on existing or known knowledge related to the event. The application of Bayes’ Theorem defines the Posterior probability \( P(\theta|D) \), where \( \theta \) represents the parameter set for given data set \( D \). The Posterior probability could be calculated as \( P(\theta|D) = \frac{P(\theta)P(D|\theta)}{P(D)} \), where the Likelihood \( P(D|\theta) \) is the probability of realizing an experimental data \( D \) given a set of parameters \( \theta \); the denominator \( P(D) \) is the probability of the evidence and could be considered as a normalizing factor; \( P(\theta) \) is the reflected known value of the considered parameters, also called as Prior. The MH algorithm is an improved algorithm based on Markov chain Monte Carlo (MCMC) simulation. The MH algorithm external limitations and targets are set or approximate before the simulation for better efficiency. The Posterior probability is then generated using the MH algorithm. First, the Prior is established for the computational model. In this nonlinear hysteresis model, the Priors with mathematical and physical constraints and Gaussian distribution are assumed for the purpose of computation. Once all the conditions are satisfied, the Prior is accepted as a scaling constant [22]. The initial vectors, \( \theta_0 \), are automatically accepted in the result. The deterministic parameters defined before from optimization approach are used as initial vectors of the MH algorithm to reduce the computational cost [29]. To make sure that the probabilistic results are reasonable, the following Likelihood function is used to examine the difference between experimental data and model prediction, \( L_i = \sum_{t=0}^{n} \sqrt{(f_0(t) - F^*(t))^2} \), where \( L_i \) means the Likelihood at \( i^{th} \) step; \( f_i(t) \) is the model predicted force at \( i^{th} \) iteration due at the time, \( t \), from 0 to \( n \), length of experiment; \( F^*(s) \) is the the MR damper force output from experiments. As iterations increases, the value of \( L_i \) increases when there is difference. A random increment draws from the previous parameter distribution that satisfies the physical and mathematical model requirement is added to parameters, which contributes to the newly proposed samples. In this study, at the first step iteration parameters distributions, deterministic values were used for the mean value; an assumed
value 0.15 were used as coefficient of variation. The new set of parameters goes through the same procedure above and $P^{(i)}$ will be examined for acceptance or rejection. If the Posterior is rejected, then the previous parameter set is reused, a different random increment is added to the previous parameter set and repeats new sample generation. If the Posterior is accepted, the new parameter is retained; then a random increment is added to the new parameter. The parameter sets should be converged when many samples were conducted; the histogram of parameters could be obtained. Fig. 1 showed a graphical flowchart of the probabilistic approach – MH algorithm as it described above.

![Flowchart of the probabilistic approach – MH algorithm](image)

Figure 2. Time History of (a) Predefined Displacement and (b) Predefined Current Input.

![Histograms of maximum damper force](image)

Figure 3. Maximum Damper Force Histogram of (a) Bouc-Wen (b) Non-Parametric Algebraic (c) Viscous Plus Dahl and (d) Hyperbolic Tangent Model.

**Performance of Probabilistic MR Damper Model**

To evaluate the effect of parameter uncertainties on the MR damper prediction, a random predefined displacement with various frequencies from 0 to 3 Hz and maximum amplitude 50 mm in Fig. 2 (a) and a current input with maximum 2.5 amps in Fig. 2 (b) [23] are used. The 200 kN MR damper is well-known for its characteristic, maximum restoring force could be generated with the maximum 2.5 amps with a short period of time. The comparison of the deterministic and
probabilistic force outputs using the parameters identified. Four identified MR damper models are compared with each other using the real-time test result data as the standard. Fig. 3 (a) - (d) and Fig. 4 (a) – (d) present the histograms, generated by using Monte Carlo simulation (MCS), of the maximum damper restoring forces and dissipated energy, respectively, throughout the displacement history for the Bouc-Wen, the non-parametric algebraic, the viscous plus Dahl, and the hyperbolic tangent models, respectively. The solid red line and the dotted blue line represent experimentally measured and deterministically predicted maximum output force, respectively.

The predicted maximum values from the models with deterministic parameters are 235.07, 264.14, 225.73, and 251.94 kN for the Bouc-Wen, the non-parametric algebraic, the viscous plus Dahl, and the hyperbolic tangent models, respectively. The predicted range of maximum values from the Monte Carlo simulations using the parameters from the probabilistic analysis varies from 150 to 250 kN for all four models. The experimental maximum force throughout the entire time history is 171.16 kN. Observation could be made that maximum damper force value from test falls in the probabilistic distribution and scattered on the left of the probabilistic mean with low probability in Bouc-Wen, non-parametric algebraic and hyperbolic tangent model (Fig. 3 (a), (b) and (d)). While the viscous plus Dahl model histogram (Fig. 3 (c)) presents the experimental maximum damper restoring force has a higher probability and scattered to the left and close to the probabilistic mean. The deterministic values for the maximum damper force can be observed to be overestimated in different models. The histogram for Bouc-Wen model shows that the probabilistic mean value reflects the deterministic indicating that the model is a stable model with many moderate correlated parameters. This observation is consistent with the findings by previous researchers. The histograms from the hyperbolic tangent and non-parametric algebraic models show obvious shifting for probabilistic mean from deterministic value to experimental data. The viscous plus Dahl model presents a high probability for both experimental data and deterministic value indicating the model could produce highly accurate maximum restoring force that is close to the reality.

The predicted energy dissipation values from the models with deterministic parameters are 316.91, 298.91, 279.19 and 268.76 kJ for the hyperbolic tangent models, the Bouc-Wen, the viscous plus Dahl, and the non-parametric algebraic, respectively. The predicted values vary from 100 to 400 kJ from 10000 Monte Carlo simulations using the parameters from the probabilistic analysis for all four models. The experimental dissipated energy throughout the entire test is about 261.14 kJ. It can also be observed that histogram of energy dissipation behaves as a normal distribution. The experimental data scattered on the left of the distribution mean; it indicates a relatively low probability in Bouc-Wen and hyperbolic tangent models (Fig. 4 (a) and (d)). Meanwhile, the viscous plus Dahl and non-parametric algebraic model distributions were presented experimental value scatted on the left of the distribution with high probability (Fig. 4(b) and (c)). Hyperbolic tangent, viscous plus Dahl, and non-parametric algebraic deterministic values are scatted on the right of the probabilistic distribution indicate overestimated dissipated energy. Bouc-Wen model histogram presents that the probabilistic mean value reflected the deterministic. This agrees well with the findings by previous researchers. The viscous plus Dahl model presents a high probability for experimental data and a moderate probability for deterministic value indicate the model could produce highly accurate maximum dissipated energy that is close to the reality.
Figure 4. Damper Dissipated Energy Histogram of (a) Bouc-Wen (b) Non-Parametric Algebraic (c) Viscous Plus Dahl and (d) Hyperbolic Tangent Model.

Figure 5. Parameters Distribution Convergence
Figure 5 (a) to (d) presents the convergence of selected model parameters including parameter $V_{\text{ref1}}$ of hyperbolic tangent model in Eq. (1a), parameter $A_5$ of Bouc-Wen model in Eq. (2c) for current input of 2.5 Amps, parameter $\rho$ of viscous plus Dahl model in Eq. (4a) and parameter $\alpha$ of non-parametric algebraic model in Eq. (3). Parameter convergence can be observed when enough samples are provided.
The convergence of the simulation are evaluated using normalized root mean square (RMS) and RMS error of energy dissipation which are defined as

\[ Err_f^{rms} = \frac{RMS(f^{exp}(t_i) - f^{sim}(t_i))}{f_{max}^{exp}} \]
\[ Err_{energy} = \frac{|E^{exp} - E^{sim}|}{E^{exp}} \]

where \( f^{exp} \) and \( f^{sim} \) are the damper forces from the physical experiment data and predicted damper model using the mean value of probabilistic parameters, respectively. \( f_{max}^{exp} \) is the maximum damper force from the physical experiment. \( E^{exp} \) and \( E^{sim} \) represent the total dissipated energy in the physical experiment and predictive model using the mean value of probabilistic parameters, respectively. Table 1 presents the two RMS errors for all four models. It can be observed that all four models have similar force RMS errors with the Bouc-Wen model for the smallest error of 28.51% and the non-parametric algebraic model for the largest error of 40.29%. The non-parametric algebraic model however shows the smallest energy error of 6.44% when compared with the other three.

<table>
<thead>
<tr>
<th>Damper Model</th>
<th>Force RMS (%)</th>
<th>Energy RMS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyperbolic Tangent</td>
<td>32.39</td>
<td>25.51</td>
</tr>
<tr>
<td>Bouc-Wen</td>
<td>28.51</td>
<td>18.38</td>
</tr>
<tr>
<td>Viscous Plus Dahl</td>
<td>37.09</td>
<td>10.57</td>
</tr>
<tr>
<td>Non-Parametric Algebraic</td>
<td>40.29</td>
<td>6.44</td>
</tr>
</tbody>
</table>

Summary, Conclusion and Future Work

Probabilistic characterization of control devices MR damper enables to quantify the uncertainties in model parameters for the better design of controllers to facilitate more realistic simulations for seismic hazard mitigation. In this study, a total of 155 experiment data sets are used to probabilistically characterize four different MR damper models parameters for a 200 kN large-scale MR damper including the hyperbolic tangent, the Bouc-Wen, the non-parametric algebraic, and the viscous plus Dahl models. Using many experimental datasets with a wide range of amplitudes, frequencies and current inputs, the statistical properties of the parameters are identified using the MH algorithm. The effects of probabilistic parameters are then evaluated using existing test results with predefined random displacement and current input. Among these four models, the viscous plus Dahl model is a relatively simple simulation model with high probability for both experimental data and deterministic value. Based on the validation test, the viscous plus Dahl model appeared to be the optimal option. Future work is necessary for further analysis and comparison between the different models. Considering more realistic comparison, the design of structural control uncertainty should be included. There are still challenges that remain in the probabilistic approach to increase the accuracy, effectiveness, and efficiency. The more efficient sampling method – Gibbs Sampling, more accurate proposed distribution, Modified Metropolis-Hastings (MMH) algorithm could be applied in the future based on the results of this study.

References


