EFFICIENCY, SUFFICIENCY, AND PREDICTABILITY OF INTENSITY MEASURES FOR PREDICTING THE CONSEQUENCES OF LIQUEFACTION ON BUILDINGS

Z. Bullock¹, S. Dashti², A. Liel³, K. Porter⁴, and Z. Karimi⁵

ABSTRACT

In this paper, we analyze the efficiency, sufficiency, and predictability of 20 ground motion intensity measures (IMs) for the purpose of predicting the settlement and residual tilt of mat-founded structures on liquefiable ground. We assess the quality of a variety of ground motion IMs at different locations, with particular attention paid to the state of practice in geotechnical earthquake engineering. The locations considered for each IM are: outcropping rock, within (or base) rock, far-field surface as estimated by equivalent-linear total stress analyses, far-field surface as estimated by dynamic effective stress analyses, and foundation as estimated by dynamic effective stress, soil-structure-interaction (SSI) analyses. Lastly, this paper quantifies the influence of the total deposit depth on the quality of each IM and location. The optimum intensity measure for predicting foundation settlement is found to be the cumulative absolute velocity (CAV) measured on the outcropping rock. Several intensity measures offer similar performance for predicting the residual tilt of foundations, but the optimum location is also outcropping rock. The quality of outcropping rock motion for predicting settlement decreases slightly with increasing deposit depth. The motion at the surface (at the foundation or in the far-field) was a worse predictor of liquefaction consequences than at the base of the soil column regardless of deposit depth, suggesting that the base motion is more representative of the excitation experienced by the complete soil-foundation-structure system.

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Efficiency, Sufficiency, and Predictability of Intensity Measures for Predicting Liquefaction Consequences

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ABSTRACT

In this paper, we analyze the efficiency, sufficiency, and predictability of 20 ground motion intensity measures (IMs) for the purpose of predicting the settlement and residual tilt of mat-founded structures on liquefiable ground. We assess the quality of a variety of ground motion IMs at different locations, with particular attention paid to the state of practice in geotechnical earthquake engineering. The locations considered for each IM are: outcropping rock, within (or base) rock, far-field surface as estimated by equivalent-linear total stress analyses, far-field surface as estimated by dynamic effective stress analyses, and foundation as estimated by dynamic effective stress, soil-structure-interaction (SSI) analyses. Lastly, this paper quantifies the influence of the total deposit depth on the quality of each IM and location. The optimum intensity measure for predicting foundation settlement is found to be the cumulative absolute velocity (CAV) measured on the outcropping rock. Several intensity measures offer similar performance for predicting the residual tilt of foundations, but the optimum location is also outcropping rock. The quality of outcropping rock motion for predicting settlement decreases slightly with increasing deposit depth. The motion at the surface (at the foundation or in the far-field) was a worse predictor of liquefaction consequences than at the base of the soil column regardless of deposit depth, suggesting that the base motion is more representative of the excitation experienced by the complete soil-foundation-structure system.

Introduction

Intensity measure (IM) selection is a particularly important and complex problem in earthquake engineering, and even more so in liquefaction engineering. Traditionally, geotechnical engineers use the peak ground acceleration (PGA) at the surface of the soil column in the free-field, obtained based on equivalent-linear total stress analyses, to evaluate the likelihood of liquefaction triggering. Recently, many studies have used efficiency, sufficiency, and predictability as metrics for assessing the quality of a given IM for use in predicting a given demand parameter (DP).

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Kramer and Mitchell [1] employed this approach to evaluate IMs for predicting liquefaction triggering, and Karimi and Dashti [2,3] used it to evaluate IMs for predicting the settlement of mat-founded structures. These new studies have shown that surface PGA, particularly when obtained from equivalent-linear site response analyses, is not necessarily an optimum IM when predicting the triggering or consequences of liquefaction.

This study expands the analysis of Karimi and Dashti [2,3] to consider a wider variety of IMs, more locations of motion, and both settlement and residual tilt of mat-founded structures. Additionally, we consider efficiency and sufficiency in both a global sense and in a scenario-specific sense in order to evaluate the quality of the IMs as a function of total deposit depth ($H$).

**Numerical database**

The numerical database used in this study has been previously described by Karimi and Dashti [2-6]. The database consists of 421 numerical models, in which 14 soil-foundation-structure system parameters were varied, analyzed under 150 ground motions using fully-coupled, effective stress, dynamic loading (for approximately 63,000 data points). These analyses were performed with the parallel version of OpenSees [7] using the pressure-dependent multi-yield (PDMY02) soil constitutive model [8,9], and consisted of 3-D finite element representations of the soil and elastic, single-degree-of-freedom oscillators for the structure.

These numerical models have a handful of key limitations. First, as continuum models, they are inherently incapable of capturing the phenomenon of soil ejecta. Second, although the models have been successfully validated using centrifuge results, the constitutive model used does not capture volumetric deformations, particularly the effect of soil sedimentation. This means that the settlements predicted numerically arise primarily from deviatoric deformation modes, and volumetric deformations may be underestimated, which become particularly important for very thick liquefiable layers. The models may not fully capture the influence of the structure’s inertia on foundation’s permanent rotation, due to a lack of interface elements between the soil and foundation. Lastly, all numerical simulations assumed horizontally homogeneous soil layers, and do not consider the influence of spatial variability in plan. See Karimi et al. [6] for the most extensive description of this numerical parametric study.

Despite these limitations, the numerical database generated by the authors provides the most extensive results to date for investigating the quality of various IMs for predicting liquefaction consequences on buildings. Case history observations rarely include recordings of motion (and those are typically not near the building), and centrifuge or 1g shake table test results do not include enough variation of parameters to analyze their influence on efficiency and sufficiency. Additionally, because centrifuge tests are often performed with artificially scaled records, sufficiency with regard to $M_W$ and $R_{rup}$ cannot be evaluated.

**Locations of motion**

This study considers five locations for evaluating the efficiency, sufficiency, and predictability of motions. First, we considered the outcropping rock (OR) motion applied to the numerical models as input. The within or base rock (WR) motion was also obtained numerically for each model configuration, which was affected by the properties of the elastic bedrock (primarily its density and shear-wave velocity) [10] as well as the geometry and dynamic properties of the overlying soil deposit.
The far-field motion as estimated by dynamic, effective stress, nonlinear, fully-coupled analyses (FF-NL) was obtained at the soil surface away from the foundation in the numerical models. The foundation motion as estimated by the same dynamic nonlinear analyses of the soil-foundation-structure system (FN-NL) was recorded at the foundation level directly. During validation, the numerical models were able to capture the accelerations at these locations well when compared to centrifuge experimental recordings [4].

The variations in soil properties and geometry in the numerical database resulted in 115 distinct soil profiles. These profiles were also analyzed in DeepSoil 6.1 [13] with the same suite of 150 outcropping rock ground motions. All profiles were discretized into uniform 1 m-thick layers. Shear wave velocity profiles were formulated following Kramer [14] and Jamiolkowski et al. [15], and the shear modulus and damping curves provided by Darendeli [16] were used. All analyses were checked for convergence. The acceleration recorded at the ground surface in these total stress, equivalent-linear analyses of 1D wave propagation was used as the equivalent-linear free-field motion (FF-EL). Figure 1 shows a schematic view of these five locations at which ground motion IMs were obtained.

![Schematic view of the considered locations of motion.](image)

**Intensity measures**

This study considers 20 IMs. Each IM reflects the amplitude, duration, and frequency content of the ground motion in different ways. Recent studies regarding liquefaction induced permanent deformations have typically recommended evolutionary IMs as the most efficient and sufficient [2,3,11]. Such IMs are called “evolutionary” because they accumulate over the course of shaking, and therefore depend on both the amplitude and duration of shaking.

The most familiar evolutionary IM is Arias intensity \( I_A \) [12]), which is defined by Eq. 1, where \( g \) is gravity, \( t_d \) is the duration of motion, and \( a(t) \) is the acceleration time history of the motion. We also consider four permutations of cumulative absolute velocity \( CAV \) that appear in the literature. Eq. 2 gives the basic definition of \( CAV \). This IM was previously selected by Karimi and Dashti [3] as the optimum IM for predicting foundation settlement due to liquefaction.

\[
I_A = \frac{\pi}{2g} \int_0^{t_d} a(t)^2 dt
\]
\[
CAV = \int_0^{t_d} |a(t)| \, dt
\]

Kramer and Mitchell [1] found that \( CAV \) above a 5 cm/s² threshold (\( CAV_5 \)) was an efficient and sufficient predictor of liquefaction triggering. \( CAV_5 \) is defined in Eq. 3, where \( \chi(a(t)) \) is a filter that is 1 if the argument exceeds 5 cm/s² and 0 otherwise.

\[
CAV_5 = \int_0^{t_d} \chi(a(t)) |a(t)| \, dt
\]

Standardized \( CAV \) (\( CAV_{STD} \)) and damage potential \( CAV \) (\( CAV_{DP} \)) were originally developed with applications to the design of nuclear plants in mind. \( CAV_{DP} \) was also recently used in Bray and Macedo’s [11] procedure for estimating the deviatoric component of foundation settlement due to liquefaction. Eq. 4 gives the definition of \( CAV_{STD} \). \( CAV_{DP} \) is equal to \( CAV_{STD} \) if pseudo-spectral acceleration in the period range from 0.1 to 0.5 seconds exceeds 0.2\( g \) and pseudo-spectral velocity exceeds 15.34 cm/s² in the period range from 0.5 to 1.0 seconds, and zero if they do not.

We also consider many peak transient IMs. These IMs depend on the amplitude of the strongest portion of shaking, and may or may not depend on frequency content. The most common of these are peak ground acceleration (PGA), peak ground velocity (PGV), and spectral acceleration at a given period (\( S_a(T) \)). We consider spectral accelerations at periods of 1 sec (previously used by Bray and Macedo [11]), the vibration period of the structure (\( T_{st} \)), the initial site period (\( T_{so} \)), and two lengthened site periods (\( 1.5T_{so} \) and \( 2T_{so} \)). These lengthened site periods reflect the softening of the profile over the course of shaking. \( S_a(T = 1.5T_{so}) \) was used in the Bray and Travasarou [17] procedure for estimating slope displacements.

The peak incremental ground velocity (\( V_{gi} \)) is a relatively new IM, and is defined as the area under the largest acceleration pulse. Jampole et al. [18] found it to be useful for predicting the largest excursion of frictional base isolators, and it was included in Bullock et al. [19] ground motion prediction equations. We include this IM because the phenomenon of liquefaction is somewhat analogous to base isolation – the liquefied material loses its stiffness, and consequently de-amplifies the high frequency ground motion above. Eq. 4 defines \( V_{gi} \), where \( t_{1,i} \) and \( t_{2,i} \) are the times when the \( i \)-th acceleration pulse begins and ends, respectively.

\[
V_{gi} = \max_i \left[ \int_{t_{1,i}}^{t_{2,i}} |a(t)| \, dt \right]
\]

The final peak transient IMs considered are the average spectral acceleration for a variety of frequency or period ranges (\( S_{a,avg}(T_1, T_2) \)). Eq. 5 defines \( S_{a,avg} \) for arbitrary values of \( T_1 \) and \( T_2 \), where \( N \) is the number of discrete values (\( T_i \)) used to represent the range from \( T_1 \) and \( T_2 \). We consider the following period ranges: \( 0.2T_{st} - 3T_{st} \), \( 0.2T_{so} - 1.5T_{so} \), and \( 0.2T_{so} - 2T_{so} \).

\[
S_{a,avg}(T_1, T_2) = \left( \prod_{i=1}^{N} S_a(T = T_i) \right)^{1/N}
\]

Lastly, we consider four IMs that do not fit neatly into the evolutionary or peak transient categories. These four IMs are related to the duration of motion, and include the 5% to 75% significant duration (\( D_{5-75} \)), the 5% to 95% significant duration (\( D_{5-95} \)), the 5% to 75% shaking intensity rate (\( SIR_{5-75} \)), and the 5% to 95% shaking intensity rate (\( SIR_{5-95} \)). These IMs are all
tied to $I_A$ – the significant durations are the time between the accumulation of the respective portions of $I_A$, and the shaking intensity rates are the rate of accumulation of $I_A$ over the course of the significant duration. These IMs may be most effective if used as part of a vector-valued IM, but we do not consider combinations of more than one IM in this study.

Quality metrics

We evaluate different IMs at the five locations described above on the basis of their efficiency and sufficiency in predicting permanent settlement and tilt of shallow-founded structures, as well as the IM’s own predictability. Efficiency is a measure of the dispersion in observations of a given demand parameter (DP) after regression using a given IM, as shown in Eq. 6. Typically, the standard deviation of the regression residuals ($r_{DP}$, defined by Eq. 7) is used, meaning that a smaller value of standard deviation reflects higher quality or efficiency. We define efficiency ($\mathcal{E}$) according to Eq. 8.

$\ln(\bar{DP}) = A + B \ln(IM)$  \hspace{1cm} (6)

$r_{DP} = \ln(DP) - \ln(\bar{DP})$  \hspace{1cm} (7)

$\mathcal{E} = \text{std}(r_{DP})$  \hspace{1cm} (8)

Sufficiency is a measure of an IM’s ability to capture the effects of intensity on a given DP. Sufficiency is measured by evaluating the bias in $r_{DP}$ on moment magnitude ($M_W$) and distance-to-rupture ($R_{rup}$), by regressing $r_{DP}$ on $M_W$ and $R_{rup}$ as shown in Eqs. 9 and 10. The coefficients from these regressions, denoted $S_M$ and $S_R$, are the sufficiency with respect to magnitude and distance, respectively. We report the absolute value of the calculated coefficients, meaning that $S$ is always positive, and smaller values are better.

$\bar{r}_{DP} = S_M(M_W)$  \hspace{1cm} (9)

$\bar{r}_{DP} = S_R(\ln(R_{rup}))$  \hspace{1cm} (10)

Global Efficiency and Sufficiency of IMs for Predicting Foundation Response

In this section, we consider $\mathcal{E}, S_M,$ and $S_R$ for evaluating the efficiency and sufficiency of different IMs in predicting foundation’s permanent settlement as calculated using the complete numerical database. We denote these as $\mathcal{E}_{G,S}, S_{M,G,S},$ and $S_{R,G,S}$ to differentiate them from the deposit depth-specific measures of efficiency and sufficiency that follow. Likewise, we denote the corresponding global values of efficiency and sufficiency for predicting residual tilt as $\mathcal{E}_{G,RT}, S_{M,G,RT},$ and $S_{R,G,RT}$. These values are representative of the effectiveness of these IMs in the context of developing probabilistic models, in which a wide variety of soil-foundation-system parameters would be included in the regression database (i.e., models are regressed using complete numerical datasets rather than on a scenario-by-scenario basis, making the global efficiency and sufficiency of an IM the most relevant metrics for selection).

Figure 2 shows $\mathcal{E}_{G,S}, S_{M,G,S},$ and $S_{R,G,S}$ for all IMs at all locations. All IMs considered
appear sufficient with regard to $M_W$ (i.e., $S_{M,S}$ is very small for all IMs). The duration-related IMs are generally the least efficient, regardless of the location. Although certain peak transient IMs outperform certain evolutionary IMs in terms of efficiency and sufficiency, all of the IMs with the best combination of efficiency and sufficiency are evolutionary. The motion at the OR and WR locations is the most efficient and sufficient, followed by FF-EL and FF-NL. The motion at FN-NL is generally less efficient than the other locations.

Figure 3 shows $E_{G,RT}$, $S_{M,GR,T}$, and $S_{R,GR,T}$ for all IMs at all five locations. Again, $S_{M,GR,T}$ is very low for all IMs, so sufficiency with regard to $M_W$ is not a concern for IM selection, and the duration-related IMs are the least efficient regardless of location. Unlike for settlement, evolutionary and peak transient IMs have similar levels of efficiency and sufficiency when predicting residual tilt. Most IMs at the FF-EL location and approximately half of IMs at the FF-NL and FN-NL locations have poor sufficiency with regard to $R_{rup}$. The OR and WR locations still offer the best combination of efficiency and sufficiency. However, the difference between FF-EL, FF-NL, and FN-NL is smaller for predicting residual tilt than settlement.

Figure 2. Efficiency and sufficiency of all considered IMs at all considered locations for predicting foundation settlement.

**Efficiency and Sufficiency of IMs as a Function of Deposit Depth**

This section evaluates trends in scenario-specific efficiency and sufficiency as a function of total deposit depth ($H$). We denote these values as $E_{SS}(H)$, $S_{M,SS}(H)$, and $S_{R,SS}(H)$ for permanent foundation settlement and $E_{S,RT}(H)$, $S_{M,RT}(H)$, and $S_{R,RT}(H)$ for foundation’s residual tilt. These values were calculated by considering only the results from three specific models, in which all parameters except $H$ were held constant to isolate the influence of $H$. This section excludes the duration-related IMs.
Figure 3. Efficiency and sufficiency of all considered IMs at all considered locations for predicting foundation settlement.

Figure 4. Scenario-specific efficiency and sufficiency as a function of total deposit depth, $H$, for foundation settlement.

Figure 4 shows these values for settlement for the OR, FF-EL, and FF-NL locations. The WR and FN-NL locations are omitted from these figures for clarity (the WR location is very similar to the OR location, and FN-NL is very similar to FF-NL). Again, all IMs at all locations are sufficient with regard to $M_W$. Efficiency and sufficiency with regard to $R_{r}^{u}$ tend to worsen for greater deposit depths ($H$) for all IMs at all locations. The efficiency and sufficiency of IMs at the FF-EL is slightly more sensitive to increasing $H$ than OR and FF-NL, particularly for peak transient IMs. This result suggests that the frequency content and duration of the outcropping rock motion is more relevant to foundation settlement than that of surficial motion in the far-field (either from equivalent-linear or nonlinear site response analyses). There are two possible explanations for this: (1) the OR or WR motion is more representative of the seismic excitation applied to the entire rock-soil-foundation-structure system than the motion at surficial locations, each of which does not capture the complete response of the system (e.g., the degree of softening and resulting
accelerations in the far-field can be different from those near the foundation; transverse foundation accelerations do not fully reflect the building’s rotational behavior nor the shear strains experienced in deeper soils due to the large degree of damping near the surface, while they are all affected by the motion at the base of the soil column); or (2) the character of motion at the surface is governed by the liquefiable layer, which is located near the surface.

Figure 5. Scenario-specific efficiency and sufficiency as a function of total deposit depth for foundation residual tilt.

Figure 5 compares $E_{S,RT}(H)$, $S_{M,S,RT}(H)$, and $S_{R,S,RT}(H)$ at different locations. For residual tilt, the differentiation among IMs in terms of efficiency and sufficiency is smaller for greater $H$ values. Also, sufficiency with regard to both $R_{rup}$ and $M_W$ improves overall for the thicker deposit. This indicates that IM selection is less important for predicting the residual tilt of foundations on thick deposits. IMs at the FF-EL location tend to be slightly less efficient than those at OR for predicting tilt on the thickest deposit, but similarly sufficient. Regardless of variation in $H$, $CAV$, $CAV_5$, and $CAV_{DP}$ at the OR location were the most efficient and sufficient IMs for predicting settlement. $CAV$, $CAV_5$, $CAV_{DP}$, $PGV$, and $V_{gi}$ were the most efficient and sufficient IMs for predicting residual tilt.

Predictability of IMs

Efficiency and sufficiency determine which IM is theoretically the best suited for predicting a given DP, but they do not provide any information regarding its practicality. For a procedure to be useful in forward-prediction, the IMs used need to be predictable as well. We measure the predictability of IMs based on the availability of models for their prediction, and on the model uncertainty of those models.

Table 1 provides a summary of the prediction uncertainty for the considered IMs, and their prediction standard deviation ($\sigma$). No models exist for directly predicting $SIR$, but one could be derived from models for $I_A$ and significant duration if a correlation model was developed for their residuals. The existing model for $CAV_{DP}$ by Campbell and Bozorgnia [20] related $CAV_{DP}$ to the geometric mean of $CAV$, meaning that its uncertainty depends on the uncertainty around $CAV$, its own prediction uncertainty, and the uncertainty around its exceedance of zero, which is related to the uncertainty around $PGA$ and $S_a$. $S_{a,avg}$ is also a special case because its median predicted value and standard deviation are functions of the predictions of $S_a$ over the given period range, their standard deviations, and the correlations among them, as discussed by Eads et al. [21]. We do not
discuss the relative total $\sigma$ for predicting $S_{a,avg}$ or $CAV_{DP}$ here.

Table 1. Summary of the predictability of the IMs considered here.

<table>
<thead>
<tr>
<th>IM</th>
<th>Prediction $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CAV$</td>
<td>0.4 to 0.7</td>
</tr>
<tr>
<td>$CAV_5$</td>
<td>0.7 to 0.9</td>
</tr>
<tr>
<td>$CAV_{STD}$</td>
<td>0.5 to 0.7</td>
</tr>
<tr>
<td>$I_A$</td>
<td>1.0 to 1.4</td>
</tr>
<tr>
<td>$PGA, S_a(T)$</td>
<td>0.3 to 0.8</td>
</tr>
<tr>
<td>$PGV$</td>
<td>0.5 to 0.8</td>
</tr>
<tr>
<td>$V_{gi}$</td>
<td>0.5 to 0.7</td>
</tr>
<tr>
<td>$D_{S-75}, D_{S-95}$</td>
<td>0.3 to 0.8</td>
</tr>
</tbody>
</table>

$CAV$ is generally the most predictable evolutionary IM. Whether $S_a(T)$ and $PGA$ are more predictable than $PGV$ and $V_{gi}$ depends on the vibration period considered and the tectonic environment. Significant durations typically have small prediction uncertainty, but were among the least efficient and sufficient IMs considered (discussed in the previous sections).

Conclusions

$CAV$ is the optimum IM for predicting foundation settlement, and $CAV$, $PGV$, and $V_{gi}$ are the optimum IMs for predicting foundation residual tilt on liquefiable soil deposits. These IMs offer the best combination of efficiency, sufficiency, and predictability for the respective DPs. The outcropping rock motion (as the excitation applied to the entire rock-soil-foundation-structure system) is the optimum location to consider, in terms of both practicality and performance. The good performance of the OR and WR locations relative to FF-EL, FF-NL, and FN-NL is likely a result of highly nonlinear and non-stationary site response in liquefiable deposits. This study highlights not only the importance of selecting an evolutionary IM that optimizes the prediction of the DP of interest, but also re-evaluating the choice of its location.

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References


interaction on liquefiable ground. *Journal of Geotechnical and Geoenvironmental Engineering*, 2015; 142 (1).


