MODELLING OF A ROCKING MASONRY WALL WITH UNBONDED POST-TENSIONING SUBJECTED TO SHAKE-TABLE TESTING

D. Kalliontzis¹, A. E. Schultz², and S. Sritharan³

ABSTRACT

This paper presents a modelling approach for estimating seismic behavior of rocking masonry walls with unbonded post-tensioning. The proposed modelling approach estimates the rocking motions of these walls including the inelastic deformations in the masonry at the wall toes and associated continuous energy dissipation in the forms of viscous damping and hysteresis action. Following previous research, which shows that these walls can experience small flexural deformations in addition to rocking, an elastic flexural mode is also included. Accordingly, a system of non-linear equations is developed and impulse-momentum equations are employed to estimate the impact energy losses in the walls. The proposed modelling approach is compared against past experiments of a rocking masonry wall with unbonded post-tensioning, which was subjected to a series of shake table motions. Modelling results show good agreement with the experimental responses, including displacement time histories, variations in post-tensioning forces and neutral axis depths, and force-displacement behavior.

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This paper presents a modelling approach for estimating seismic behavior of rocking masonry walls with unbonded post-tensioning. The proposed modelling approach estimates the rocking motions of these walls including the inelastic deformations in the masonry at the wall toes and associated continuous energy dissipation in the forms of viscous damping and hysteresis action. Following previous research, which shows that these walls can experience small flexural deformations in addition to rocking, an elastic flexural mode is also included. Accordingly, a system of non-linear equations is developed and impulse-momentum equations are employed to estimate the impact energy losses in the walls. The proposed modelling approach is compared against past experiments of a rocking masonry wall with unbonded post-tensioning, which was subjected to a series of shake table motions. Modelling results show good agreement with the experimental responses, including displacement time histories, variations in post-tensioning forces and neutral axis depths, and force-displacement behavior.

Introduction

Significant attention has been given to the use of unbonded post-tensioning (UPT) to develop rocking structural members for seismic resilience (e.g., [1-17]). These members are designed to respond primarily by rocking when excited laterally by seismic motions and they re-center with small residual deformations by means of the restoring force provided by UPT. The seismic behavior of rocking members has been investigated in numerous experimental research studies as referenced above, but little attention has been given to dynamic modelling of their seismic responses. For example, most of the previous researchers developed models of rigid members to characterize rocking motions (e.g., [18-26]), which may not be adequate to explain the behavior of rocking structural members that deform inelastically in the critical regions at the base, while they may also experience flexure within the members (e.g., [3]). Even when flexure was included

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in the models, the member’s base was again modeled as rigid, assuming that during rocking motions, the member contacts its foundation at a single point [27-29]. However, experiments of rocking structural members have indicated that these members contact their foundation over a finite length (e.g., [30]).

This paper addresses these concerns for the case of rocking masonry walls, and develops a modelling approach to estimate their behavior under seismic excitations. A system of non-linear equations of motion is developed, accounting for the rocking and flexure modes, which dominate the responses in these walls, as shown previously [3 and 17]. Combining impulse-momentum equations with the Modified Simple Rocking Model (MSRM), proposed in [30], the impact energy losses in these walls are computed. Finally, fiber-element modelling is used to estimate the behavior of masonry at the wall base, including the hysteretic energy dissipation in the wall.

**Modelling Approach**

**Fig. 1** describes schematically the developed modelling approach using a wall of height $H_w$, length $L_w$, and thickness $t_w$. A beam is attached on the top face of the wall, providing additional mass, with height $H$, and length $L_r$. The wall is anchored to the foundation using UPT bars, which are located at distance $n_{art}$ from the wall centerline. The wall motion is described using three degrees of freedom: $\theta, u$, and $v$, which represent the rotational, flexural, and vertical motions of the wall, respectively. All three degrees of freedom are estimated with respect to the inertia frame located at the middle of the foundation base, $N\{x,z\}$. A moving frame, $E\{\xi, \eta\}$, is located at the middle of the bottom face of the wall, as shown in **Figs. 1b** and **1c**.

![Diagram](image)

(a) Initial state  (b) Rocking  (c) Flexure

**Figure 1.** A masonry wall responding by rocking and flexure.
All inelastic deformations in the masonry are assumed to occur at and nearby the wall base, which is simulated by discretizing the wall end section into fiber-elements (i.e. where the \( j \)th element is denoted as \( e_j \) in Fig. 1b). All fiber-elements located within the neutral axis depth (NAD) of the wall experience a linear strain distribution, as indicated in Fig. 1b. No strain is imposed on the elements that lift off at the foundation base, except any residual strains that have been induced in preceding responses. Inelastic deformations in the wall are assumed to extend vertically up to a height of \( Z_{inelastic} \), which is measured from the foundation base upwards. More information on estimating \( Z_{inelastic} \) for rocking masonry walls as well as details on the above-described fiber-element model can be found in [17].

Since the rocking masonry walls have been shown to develop small flexural deformations (e.g., [3, 17]), the overall flexure mode, shown in Fig. 1c, is assumed to be elastic. Moreover, shear deformations are not accounted in the proposed modelling, as previous research has shown that their percent contribution to the total wall displacements is below 3% ([3, 17]).

**Equations of Motion**

For small lateral drifts at the wall top (e.g., below 3%), the position of a point of the wall with respect to the inertia frame \( N \{ x, z \} \) can be described as follows:

\[
\begin{align*}
x &= (u + \eta)\cos \theta + (\xi - nu')\sin \theta \\
z &= v - (u + \eta)\sin \theta + (\xi - nu')\cos \theta
\end{align*}
\]  

(1)

where \( u, u' \) denote the flexural displacement of the wall and its first-order derivative with respect to \( \xi \), respectively. Variable \( u \) is estimated assuming the shape function \( \psi (\xi) \):

\[
\psi (\xi) = \frac{3\xi^2}{2H_w} - \frac{\xi^3}{2H_w}
\]  

(2)

where \( \psi (\xi) \) is defined for \( 0 \leq \xi \leq H_w \). Assuming that the masonry wall is subjected to a horizontal ground excitation with acceleration time history of \( \ddot{u}_g (t) \), the equations of motion of the wall can be derived using extended Hamilton’s principle [31]:

\[
\begin{pmatrix}
-1_q (I_z + I_x) & -1_q (I_z + I_x) & 1_q \cos \theta + I_x \sin \theta & 1_q \cos \theta + I_x \sin \theta & \dot{\theta} & -1_q (I_z + I_x) \dot{\theta} \\
-1_q (I_z + I_x) & -1_q (I_z + I_x) & I_x \sin \theta & I_x \sin \theta & \dot{\theta} & (I_z + I_x) \dot{\theta} \\
1_q \cos \theta + I_x \sin \theta & I_x \sin \theta & -M \ddot{\theta} & -I_x \ddot{\theta} & \dot{\theta} & \dot{\theta} \\
-1_q \cos \theta - I_x \sin \theta & -I_x \sin \theta & \ddot{u}_x + \sum_{j=1}^{N_e} C_{n_j} \frac{n_j x_j}{\cos \theta} - \sum_{j=1}^{N_e} \frac{F_{enj}}{\cos \theta} (\sin \theta - n_j n_{z_j}) & -1_q & \ddot{\theta} & -c_{flexx} \ddot{\theta} \\
-1_q \cos \theta - I_x \sin \theta & -I_x \sin \theta & \ddot{u}_x + \sum_{j=1}^{N_e} C_{n_j} \frac{n_j x_j}{\cos \theta} - \sum_{j=1}^{N_e} \frac{F_{enj}}{\cos \theta} (\sin \theta - n_j n_{z_j}) & -1_q & \ddot{\theta} & -c_{flexx} \ddot{\theta}
\end{pmatrix}
\]  

(3)
where $I_{c-g}$ are constant parameters describing inertia and stiffness terms, as defined in the Appendix; $g$ denotes the acceleration of gravity; $M$ and $M_s$ are the masses of the wall and imposed seismic mass, respectively; $C_{mj}$ is the compressive force exerted vertically by the $j^{th}$ fiber-element of the wall end section; $F_{PTi}$ is the force exerted by the $i^{th}$ UPT bar along the axis of the bar; $n_{mj}$ and $n_{PTi}$ denote the $\eta$-axis coordinates of the $j^{th}$ fiber-element and $i^{th}$ UPT bar, respectively; $N_{fe}$ and $N_{PT}$ denote the total numbers of fiber-elements and UPT bars in the wall, respectively; $c_{\text{interface}}$ and $c_{\text{flexure}}$ denote the damping coefficients corresponding to the viscous damping forces exerted by a fiber-element and the flexural mode, respectively. These coefficients can be estimated as follows:

\[
c_{\text{interface}} = \frac{2\zeta_{\text{interface}} E_m L_w Z_{\text{elastic}}}{N_{fe} (M + M_s)}
\]

\[
c_{\text{flexure}} = \frac{2\zeta_{\text{flexure}} I_c (I_s + I_l)}{I_k}
\]

where $\zeta_{\text{interface}}$ and $\zeta_{\text{flexure}}$ denote the fractions of critical damping associated with energy dissipation in the form of viscous damping, attributed to deformations at the wall-to-foundation interface and flexural deformations, respectively. Also, $E_m$ denotes the modulus of elasticity of masonry that can be estimated per MSJC [32].

**Estimation of Impact Response**

Rocking members dissipate part of their kinetic energy while making impact on their base. Housner [18] developed the Simple Rocking Model (SRM) to estimate the energy losses associated with these impacts in the form of coefficient of restitution, $r$. More recently, the MSRM [30] was introduced to improve estimates for these losses especially in rocking members with height-to-length ratios below 20. The MSRM’s equation for $r$ is detailed as follows:

\[
r = \left[ \frac{1 + MR^2 (1 - (\sin \alpha)^2 (1 + k^2))}{1 + MR^2 (1 - (\sin \alpha)^2 (1 - k^2))} \right]^{1/2}
\]

where $I_{cm}$ denotes the moment of inertia of the rocking member with respect to its center of mass; $R$ is the distance of the member’s center of mass from its bottom corner; $k$ is a dimensionless factor, with a suggested value of $k = 0.72$ [30]; and $\alpha$ is computed as shown below:

\[
\alpha = \tan^{-1} \left( \frac{L_w / 2}{Z_{cm}} \right)
\]
where \( Z_{cm} \) is the height of the member’s center of mass with respect to the foundation base. Based on the MSRM, when a rocking member makes impact with its foundation, the associated impulses are concentrated at the location of its rotation center just after the impact; this center is assumed to be at \( kLw/2 \) from the centerline at the member’s base. Using this assumption, the proposed modelling computes the wall responses just after an impact by applying impulsive forces at point O of the wall base, as shown in Fig. 2. It can be assumed that the impact event occurs when point O just contacts the foundation base during re-centering of the wall. To estimate the impact response of the wall, Eq. 5 is combined with the horizontal, vertical, and rotational impulse-momentum equations. For the case in Fig. 2 (i.e. \( \theta(t) < 0, \dot{\theta}(t) > 0 \)), the response just after an impact is estimated using Eq. 7:

\[
\begin{pmatrix}
\dot{\theta}^+
\\dot{\theta}^-
\end{pmatrix}
= \begin{pmatrix}
I_x \cos \theta - I_y \sin \theta & 0 & -1 & 0
d - I_y \sin \theta & (M + M_t) & 0 & -1
d & 0 & R_s \cos(\alpha + \theta) & R_s \sin(\alpha + \theta)
d & 1 & 0 & 0 & 0
\end{pmatrix}^{-1}
\begin{pmatrix}
[I_x \cos \theta - I_y \sin \theta] \dot{\theta}^-
d - I_y \sin \theta & (M + M_t) \dot{\theta}^-
d & 0 & R_s \cos(\alpha + \theta) & R_s \sin(\alpha + \theta)
\end{pmatrix}
\]

(7)

where the superscripts ‘+’ and ‘–’ denote the wall responses just after and just before the impact, respectively; and \( R_s \) is the distance of the wall’s center of mass from point O at the wall base.

Figure 2. Impact event with vertical, \( I_z \), and horizontal, \( I_x \), impulsive forces acting at O.

**Comparisons with Experimental Responses**

**Description of Experiments**

To validate the proposed modelling approach, the shake-table tests of a rocking partially-grouted concrete masonry wall with UPT (referred to as Wall 1 here), which was tested by Wight et al. [33], are used. In the first stage of the test program by Wight et al., Wall 1 was connected with a second masonry wall (Wall 2) using a vertical shrinkage control joint. The jointed wall system was subjected to fourteen shake table tests of various intensities with the prestressing forces in Wall 1 varying from about 39.4 kN to 77.5 kN. As part of this system, Wall 1 experienced top lateral
drifts of up to 1.23%.

Next, Wall 1 alone was subjected to a series of twenty two shake table tests. From this test series, tests #7-9 were selected to validate the proposed modelling approach in this paper. The main reason for this selection is that these three tests recorded significant variations in the maximum lateral drift experienced by the wall per test (i.e. from 0.35 to 1.40%), including the overall maximum drift recorded from all the tests. Moreover, a free vibration test was also included (test #9), which was used to characterize the natural decay of motion of the wall. Design parameters of Wall 1 are presented in Figure 3.

![Figure 3. Wall 1 subjected to shake table motions by Wight et al. [33].](image)

### Comparisons

As Wall 1 had already been subjected to significant lateral drifts prior to test #7, it was hypothesized that some degradation should have occurred in the masonry of the wall base. To estimate this degradation, a lateral drift history that captured all maximum positive and negative lateral drifts experienced by this wall in each test prior to test #7 was applied to the developed wall model in a static manner using displacement control. The corresponding lateral drift history is presented in Fig. 4 together with the residual strains along the wall base, as computed in the model.

![Figure 4. a) Sequence of maximum positive and negative wall drifts for Wall 1 prior to test #7; and b) Computed residual strains along the wall length prior to test #7.](image)

<table>
<thead>
<tr>
<th>Mass of Wall</th>
<th>577.6 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of Added Mass</td>
<td>1,438.5 kg</td>
</tr>
<tr>
<td>Total Mass of Rocking System</td>
<td>2,016.1 kg</td>
</tr>
<tr>
<td>Wall Height</td>
<td>2,438 mm</td>
</tr>
<tr>
<td>Wall Width</td>
<td>1,016 mm</td>
</tr>
<tr>
<td>Wall Thickness</td>
<td>143 mm</td>
</tr>
<tr>
<td>Tendon Cross-Sectional Area</td>
<td>177 mm²</td>
</tr>
<tr>
<td>Initial Post-Tensioning Force</td>
<td>75 kN</td>
</tr>
</tbody>
</table>
Incorporating the residual strain estimates into the initial conditions of the wall, the wall model was subjected to the shake table motions of tests #7-9. It was assumed in the model that $\zeta_{\text{flexure}} = \zeta_{\text{interface}} = 5\%$, which are reasonable estimates of damping ratios associated with the dynamic behavior of masonry wall systems, based on previous research studies (e.g., [34]). Because of space limitations, only the responses for tests #7 and #8 are presented in this paper.

**Fig. 5** presents the lateral drift vs. time responses of the wall as recorded experimentally and as computed from the proposed model, showing good agreement. Good agreement between the model and experiments are also seen for the post-tensioning force and neutral axis depth (NAD) vs. lateral drift responses, which are presented in **Fig. 6**. The model results are shown to deviate from the experimental NAD only for very small lateral drifts, below 0.2%. It is unclear if these deviations are real or if they can be attributed to noisy experimental data during the impacts; the significant drop observed in the experimental NAD implies that the wall lost all contact with its base at lateral drifts below 0.1%.

![Figure 5](image5.png)

**Figure 5.** Comparisons of experimental wall drift vs. time responses with the proposed modelling results for tests #7 and 8.

![Figure 6](image6.png)

**Figure 6.** Comparisons of experimental post-tensioning force and neutral axis depth in test #8 with the proposed modelling results.

Finally, **Fig. 7** presents experimental peak drifts and associated lateral forces in the wall as estimated in [33], comparing them with the corresponding model estimates. Good agreement is achieved.

![Figure 7](image7.png)
Conclusions

This paper introduces a modelling approach to capture the behavior of rocking masonry walls with unbonded post-tensioning under seismic excitations. A system of non-linear equations of motion is developed, which accounts for rocking and flexural motions of these walls. The model also accounts for continuous energy dissipation in the walls, in the forms of viscous damping and hysteresis at the wall base as well as viscous damping within the walls due to flexure. Moreover, impact energy dissipation is accounted by introducing impulsive forces at the wall base during the impact events.

To validate the modelling approach, past experiments of a rocking concrete masonry wall with unbonded post-tensioning subjected to shake table motions were employed. In comparison with these experiments, the model was able to achieve good comparisons all around, including the post-tensioning force responses, neutral axis depths at the wall-to-foundation interface, lateral-force responses of the wall, and displacement vs. time histories.

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Appendix

This section provides the equations for computing the constant parameters named as $I_{i-9}$ in the main body of the paper. They are detailed as follows:

$$I_i = (\rho_n L_n) \int_0^{H_n} \psi^2 d\xi + M_s$$  \hspace{1cm} (A-1)

$$I_z = (\rho_n L_n) \int_0^{H_n} \xi \psi d\xi + M_r \frac{H_i H_n}{2} \left[ \left( \frac{1 + \frac{H_n}{H_i}}{H_r} \right)^2 - \left( \frac{H_n}{H_r} \right)^2 \right]$$  \hspace{1cm} (A-2)
\[ I_3 = (\rho_m, L_w) \int_0^H \psi \, d\bar{\xi} + M_s \quad (A-3) \]
\[ I_4 = \left( \frac{\rho_m, L_w}{12} \right) \int_0^H (\psi')^2 \, d\bar{\xi} + M_s \left( \frac{L_w^2}{12} \int_0^H \left( \frac{3}{2H_w} \right) \right) \quad (A-4) \]
\[ I_5 = M \left( \frac{L_w^2}{12} + \frac{H_w^2}{3} \right) + M_s \left( \frac{L_w^2}{12} + \frac{H_w^2}{3} \left( \frac{1 + H_w}{H_w} \right) - \left( \frac{H_w}{H_w} \right)^2 \right) \quad (A-5) \]
\[ I_6 = \left( \frac{\rho_m, L_w}{12} \right) \int_0^H \psi'' \, d\bar{\xi} + M_s \left( \frac{L_w^2}{12} \int_0^H \left( \frac{3}{2H_w} \right) \right) \quad (A-6) \]
\[ I_7 = H \int_0^H E_s L_w (\psi'')^2 \, d\bar{\xi} \quad (A-7) \]
\[ I_8 = \int_0^H \psi'^2 \, d\bar{\xi} \quad (A-8) \]
\[ I_9 = M \frac{H_w}{2} + M_s \frac{H_w}{2} \left[ \left( \frac{1 + H_w}{H_w} \right)^2 - \left( \frac{H_w}{H_w} \right)^2 \right] \quad (A-9) \]

where \( \rho_m \) is the density of the masonry assembly; \( \psi', \psi'' \) are the first- and second-order derivatives of the shape function \( \psi \) with respect to \( \bar{\xi} \); and \( I_m \) is the moment of inertia of the wall’s cross-section.

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