EVALUATION OF A POLYNOMIAL CHAOS EXPANSION APPROACH FOR STEEL MRFS WITH VISCOUS DAMPERS

Y. Xu\textsuperscript{1}, C. Chen\textsuperscript{2} and J. Liang\textsuperscript{3}

\textbf{ABSTRACT}

Numerical simulation plays an important role for seismic risk assessment of civil engineering infrastructures under earthquakes. Nonlinear Time History Analysis (NLTHA) provides a common solution for system-level structural response prediction however involves significant computational burden especially for complex structures. How to efficiently and accurately address structural response prediction presents a challenge for the community. In this study, the metamodeling method of Polynomial Chaos Expansion (PCE) is evaluated for response prediction of structures under earthquakes. An OpenSees finite element model is integrated with meta-modeling of a steel moment-resisting frame with viscous dampers. The procedure of PCE analysis is presented and the accuracy of the PCE approach is evaluated through the comparison with NLTHA results. Effects of various factors are investigated on the performance of the PCE technique including the training data, the number of inputs, and the order of PCE. Results from this study demonstrate that the PCE technique provides a potential technique for effectively and efficient structural response prediction.

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ABSTRACT

Numerical simulation plays an important role for seismic risk assessment of civil engineering infrastructures under earthquakes. Nonlinear Time History Analysis (NLTHA) provides a common solution for system-level structural response prediction however involves significant computational burden especially for complex structures. How to efficiently and accurately address structural response prediction presents a challenge for the community. In this study, the metamodeling method of Polynomial Chaos Expansion (PCE) is evaluated for response prediction of structures under earthquakes. An OpenSees finite element model is integrated with metamodeling of steel moment-resisting frame with viscous dampers. The procedure of PCE analysis is presented and the accuracy of the PCE approach is evaluated through the comparison with NLTHA results. Effects of various factors are investigated on the performance of the PCE technique including the training data, the number of inputs, and the order of PCE. Results from this study demonstrate that the PCE technique provides a potential technique for effectively and efficient structural response prediction.

Introduction

In modern engineering, numerical simulation, also known as computer model based simulation, plays an important role in research and design process. For different disciplines (e.g. earthquake engineering, biochemical engineering, mechanical engineering), the computer model based simulation allows successful response prediction of systems based on the mathematical functions developed to describe the system behavior. For example, from the models with less complexity, Cooper [1], and Cooper and Stroup [2] developed computer models to describe the evolution of fire within a confined area. Chang et al. [3] developed a finite element model (FEM) to describe the stress in the upper medial femur and the deformation of implants of a replaced joint. Montgomery and Truss [4] used a computer model to determine the failure depth of pockets in sheet metal. Ichimura et al. [5] developed a numerical model to reproduce urban earthquake response of Tokyo city to model involves extreme complexity. These applications demonstrate the value of computer model based numerical simulation. However, numerical simulations can be time-consuming when the model involves significant complexity or requires large amounts of

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analyses for different scenarios.

One approach for reducing this burden is the integration of surrogate modeling (i.e., metamodeling) techniques, which offer an approximate functional relationship between the input of the adopted models and the resultant structural response that replaces the initial, high-fidelity model. Response surface method (RSM) was shown to provide an efficient and effective approximation for structural reliability problems [7]. Sacks et al. [8] introduced the Kriging method for engineering design problems and it has been used by later researchers. Gidaris et al. [9] applied Kriging to approximate the relationship between structural response and structural and earthquake parameters for the seismic risk assessment. Dubourg et al. [10] and Zhao et al. [11] use the Kriging approach to solve problems of design optimization for reliability where these problems have high-cost performance models and are not affordable for simulation-based analysis. The PCE method [12] has also been used for reducing the computational time [13]. Spiridonakos and Chatzi [14] proposed the application of PCE to represent the response of the nonlinear auto-regressive model with eXogenous input form. Sudret and Mai [15] applied PCE to predict the time history of the inter-story drifts of a linear elastic moment resistant frame under earthquakes for fragility analysis. These studies have shown that the metamodeling methods can be applied for predicting the system responses with a reduced computational burden compared with direct computer model simulation. In this study, the polynomial chaos expansion (PCE) is evaluated for response prediction of nonlinear building structures under earthquakes.

**Meta-modeling using Polynomial chaos expansion**

Assume that the system output is described by a numerical model $y(x)$, and the PCE representation of the model $y(x)$ can be written as below [16]:

$$y(x) \approx y^{pc}(x) = \sum_{a \in A} c_a \psi_a(x)$$

(1)

where $y$ is the finite element model; $x$ is the input random variables following predefined distributions; $y^{pc}(x)$ is the PCE prediction of system output; $c_a$ is the PC coefficients; $\psi_a(U)$ is the selected multivariate orthonormal polynomials associated with the distributions of input random variables, and $A \in \mathbb{N}$ is the set of selected multi-indices of multivariate orthonormal polynomials. Table 1 presents the orthonormal polynomials for different types of distributions [17]. A truncated series of polynomials in Eq. (1) are often used and the number of coefficients for truncated PCE polynomials with total degrees of $S$ can be calculated as:

$$P = \frac{(R+S)!}{R!S!}$$

(2)

where $P$ is the number of total PCE coefficients; $R$ is the number of random inputs, and $S$ is the order of PCE.

<table>
<thead>
<tr>
<th>Type of variable</th>
<th>Orthogonal polynomials</th>
<th>Type of variable</th>
<th>Orthogonal polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>Legendre</td>
<td>Gamma</td>
<td>Laguerre</td>
</tr>
<tr>
<td>Gaussian</td>
<td>Hermite</td>
<td>Beta</td>
<td>Jacobi</td>
</tr>
</tbody>
</table>
The PCE coefficients can be computed by nonintrusive methods such as the projection method and the regression method. It can be observed from Eq. (2) that the number of PCE coefficients can increase dramatically with the increase of the number of input variables and the PCE order, thus resulting in excessive computational demand. A Least Angle Regression (LAR) algorithm based sparse PCE approach [18] is applied, where only part of the PCE coefficients are computed with the rest of the coefficient assigned zero value. In this study, the Matlab [19] based framework Uncertainty Qualification Laboratory (UQLab) [20] is used to conduct the sparse PCE analysis.

**Steel MRF with Viscous Dampers**

**Prototype Steel MRF with Viscous Dampers**

A five-story three-bay steel MRF with viscous dampers [21] is selected in this study as shown in Fig. 1. This building is assumed to be located at downtown Los Angeles with site class of D. The seismic mass of the building is the sum of dead load and 25% of the live load with uniform 100 ksf floor dead load and 80 ksf live load. All steel members have Young’s modulus of 29,000 ksi, and yield strength of 36 ksi for beams and 50 ksi for columns. Inherent Rayleigh damping of 5% of critical was assigned based on the first and third elastic modes of vibration of the structure. The nonlinear viscous fluid dampers in this study have the damping coefficient of 82.8 lbf(s/in)^0.5, and the velocity exponent of 0.5. Table 2 presents the member sections for the beams and columns as well as the number of viscous dampers for each story. The fundamental period of the MRF is 1.54 second.

![Scheme of the OpenSees model](image1)

**Figure 1.** Scheme of the OpenSees model.

**Figure 2.** Time history of the ground acceleration.

The finite analysis software Open System for Earthquake Engineering Simulation (OpenSees) [22] is used to model the MRF with viscous dampers for structural response prediction in this study. The inelastic response of material was simulated by the use of nonlinear fiber hinges at the framing members, which has the hysteretic behavior governed by the modified Ibarra-Krawinkler deterioration model [23]. The rigid zones are applied to the beam-column connections to prevent panel shear deformation and yielding under earthquake loading to
represents the scenario that all hysteretic energy is dissipated through plastic hinging in the beams and columns. Lean columns made from axially rigid beam pinned at both ends are placed parallel to the frame with each floor under the constraint to the floor translational displacement and the gravity loading was applied to the non-seismic elements. Rigid-end offsets were specified at the end of the frame members to account for the actual size of the members at the beam-column joints.

The 1994 Northridge earthquake recorded at Beverly Hills-12520 Mulhol station is used in this study with a peak ground acceleration of 0.516g as shown in Figure 2. The ground motion has a total duration of 29.95 seconds with an incremental of 0.01s, and it has been cut at the end to make it 29s while the ground acceleration value is almost zero.

### Table 2. MRF member section property.

<table>
<thead>
<tr>
<th>Beam Section</th>
<th>Column Section</th>
<th>Number of viscous damper</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th floor</td>
<td>W16X57</td>
<td>5th floor</td>
</tr>
<tr>
<td>4th floor</td>
<td>W16X57</td>
<td>4th floor</td>
</tr>
<tr>
<td>3rd floor</td>
<td>W18X57</td>
<td>3rd floor</td>
</tr>
<tr>
<td>2nd floor</td>
<td>W18X57</td>
<td>2nd floor</td>
</tr>
<tr>
<td>1st floor</td>
<td>W18X57</td>
<td>1st floor</td>
</tr>
</tbody>
</table>

Meta-modeling for Structural Response at Each Time Instant

The selected ground motion record can be considered as a vector $G_{acc,i}$ of 2900 elements at the time instant of $t_i$. Multiple vectors $X^n$ are generated where $n$ is the sampling number, and each vector $X^n$ has 2900 independent Gaussian random variables $X^n_{i}$ with the mean value of 1 and standard deviation of 0.2. By multiply $G_{acc,i}$ and corresponding $X^n_{i}$, a set of new ground accelerations are generated, and then the ground motions have been scaled 10 times larger. The final product of ground motions will have similar spectral content as shown in Figure 3. The OpenSees model is then subjected to the newly generated ground accelerations, and the roof displacements are collected for the following metamodeling process. Fig. 4 shows the hysteresis behavior of the damper at the third floor subjected to one of the modified ground accelerations.

When forming the meta-model, a PCE meta-model has to be constructed for every time instant in order to use PCE to reproduce structural response for the entire duration of the ground motion. For the $n^{th}$ modified ground motion, not all $X^n_{i}$ have the influence on the roof displacement at time instant $t_i$ since the inter-story drift do not depend on the $X^n_{j}$, $j > i$, after the time instant $t_i$. Therefore, only the $X^n_{i}$ before the time instant $t_i$ will be used to obtain the PCE model. After the PCE at $t_i$ is established, a new set of input vectors $X^n$ are generated to validate the meta-model. The new story drift between the third and second stories are then calculated using both the OpenSees model and meta-model based on new ground motions. The comparison of the results is presented in Fig. 5. It can be observed that the PCE models have a good accuracy at different time instants. Similar observations can also be made for the rest of the ground motion.
Figure 3. Pseudo acceleration spectrum.
Figure 4. Hysteresis behavior of damper at third floor.

Figure 5. Comparison of roof displacements at time instants of (a) 4s, (b) 8s, (c) 12s, (d) 16s, (e) 20s and (f) 24s.

**Evaluations of Sparse PCE Modeling for MRF**

**Sampling Number, and order of Sparse PCE**

The optimal sampling number for regression method [24] can be calculated as $P \ast (R - 1)$ where $P$ is the number of coefficients and $R$ is the number of input variables. Considering the fact that the ground motion inputs are used as input random variables, computational efforts significantly increases with the increase of the sampling number and the order of PCE analyses. Convergence tests were conducted to evaluate the performance of the sparse PCE models with the different values of sampling number, and the order of PCE analyses. The accuracy of the PCE meta-model is quantitatively measured through the LOO error. Fig. 6 supports that the accuracy of PCE models increase with the increase of sampling number.
Lucor and Karniadakis [25] indicated that increasing the order of the PCE meta-model will increase the accuracy. However, when the sample number is limited, higher order sparse PCE could lead to larger LOO error. Fig. 5 shows the LOO error for 1st and 2nd order PCE meta-model at 4s. It can be observed that with a small amount (e.g. 300) of sampling numbers, the 1st order sparse PCE has a better performance compared to the 2nd order PCE, which has higher LOO error value; while with the increasing of sampling number, 2nd order PCE can have better accuracy. Also it can be observed that there are some fluctuations while the sampling number is increasing, and the reason is that the performance of PCE based on the sampling points been generate. Based on the convergence test, the sampling number of 600 and the 1st order PCE were selected for producing full time history prediction.

Figure 6. LOO errors for different sample number at time instant of 4s.

Structural Response Prediction

With the first order PCE and sampling number of 600, a total of 2900 PCE meta-models were conducted for each time instant of the selected ground motion. After that the PCE meta-model is established, a new set of $X$ is generated to validate the meta-models. The comparison between the transient FEM analysis results and the PCE meta-models prediction is presented in Fig. 7. Good agreement can be observed implying that the sparse PCE technique can be a potential tool for predicting the roof displacement. Some limitations of the sparse PCE approach, however, are observed from the computational simulation as well. Fig. 8 shows that the LOO error of sparse PCE models also increases with the time. The computational burden of PCE approach depends on the length of time duration for prediction. This means that the PCE approach can be costly if the prediction time duration is long. Moreover, the PCE models show good performance for the ground motions with similar spectral characteristics with those used for establishing PCE models. Further study is necessary to explore its accuracy for ground motions with different spectral characteristics.
Sparse PCE is used to reproduce the time history of a steel moment frame with viscous dampers under earthquake in this study. The outputs from transient FEM analysis using OpenSees are utilized to evaluate the performance of the meta-models established through sparse PCEs. The PCE coefficients, sampling number and PCE order are determined based on the comparison between meta-model outputs and FEM analysis. Convergence tests are conducted to select appropriate sampling number, and PCE order to balance between efficiency and accuracy. The results show that the increase of the sampling number leads to a more accurate prediction of system responses. The comparison of structural response with the OpenSees model output over the entire ground motion duration shows that PCE technique could be a potential tool for nonlinear structural response prediction while good agreements can be overserved from OpenSees model outputs and PCE meta-model outputs under the ground motion inputs with similar spectral characteristics. There, however, are limitations to this approach while every time instant requires a PCE model, which may lead to heavy computational burden when the prediction duration is long. The PCE models also show the trend of LOO errors increasing with the time. Future studies will be conducted on using different sampling methods to reduce the computational burden, increase the suitable range of ground motions, and control the cumulative error.

References