MODELING SLABS FOR SEISMIC DESIGN OF CONCRETE BUILDINGS

Ricardo E. Barbosa\textsuperscript{1} and Jose J. Alvarez\textsuperscript{2}

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Modeling Slabs for Seismic Design of Concrete Buildings

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The limitations of the common practice to design reinforced concrete buildings assigned to seismic design categories (SDC) D to F, based on the results of the analysis of a mathematical model in which slabs are modeled using shell elements, are evaluated. Although slabs are flexural elements and modeling them with shell elements improves the representation of their behavior under gravity loads, in the seismic analysis, such modeling alters the representation of the seismic force-resisting system. Modeling slabs as shells, implies that they also contribute to resist seismic forces through out-of-plane bending, which results in a significant reduction of the computed story drift ratios and the required reinforcement of the vertical elements of the seismic force-resisting system. To rely on such reductions, it would be required to ensure that slabs can actually resist those seismic bending moments and shear forces according to the special provisions for earthquake resistance design, which is not feasible in practice. It is for this reason that slabs are not considered part of the seismic force-resisting system of buildings SDC D-F, and therefore, they must be modeled as membranes (diaphragms), which just distribute the inertial forces of the floor to the vertical elements of the seismic force-resisting system, through in-plane stresses.

Introduction

Seismic design of reinforced concrete buildings assigned to seismic design categories D to F, is often based on the results of the analysis of a mathematical model in which slabs are modeled using shell finite elements instead of membrane elements. The use of shell elements to model slabs is becoming popular because first they are the recommended type of elements in the documentation of widely used structural software [1]. Secondly, because in such software the procedure used to transfer slab loads to beams and walls is more accurate when slabs are modeled as shell elements, especially in cases of complex geometry. Lastly, because the use of shell elements supposedly results in a more economical design.

The use of shell elements to represent slabs in the lateral load analysis, however, implies that the

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slabs contribute to resist inertial forces through out-of-plane bending. The rotational restrain created by the moment resistance of the slabs, may drastically reduce the computed story drifts and the computed amount of reinforcement required by the vertical elements of the seismic force-resisting system. Nevertheless, to be able to account on such reductions, it would be required to ensure that slabs can in fact resist those bending moments and shear forces produced by the earthquake, according to the special seismic provisions for flexural members. The above requirement is not feasible in practice, which is the reason why slabs cannot be considered part of the seismic force-resisting system of buildings assigned to categories D to F [2].

In this paper, after a brief description of the types of elements available to model slabs and walls, the error introduced in the seismic analysis and design of buildings by modelling slabs as shell elements is demonstrated through an application example.

**Plane Finite Elements**

The types of finite elements available to model structural elements such as slabs and walls are shown in Fig. 1. They are Membrane, Plate and Shell elements. The Membrane element is a diaphragm type of element, with in-plane stiffness only, restricting in-plane displacements \( U_1 \) and \( U_2 \) and rotation about the normal axis \( R_3 \). The element does not constraint normal displacement \( U_3 \) nor rotations \( R_1 \) or \( R_2 \). As shown in Fig. 1a, the internal resisting forces are in-plane forces \( F_{11} \) and \( F_{22} \) and in-plane shear force \( V_{12} \).

The Plate element is a flexural element, commonly used for the gravity analysis of slabs. The element restricts normal displacement \( U_3 \) and rotations about in-plane axes \( R_1 \) and \( R_2 \). As shown in Fig. 1b, the internal resisting forces of the element are transverse shears \( V_{13} \) and \( V_{23} \), flexural moments \( M_{11} \) and \( M_{22} \), and torsional moment \( M_{12} \).

The Shell element is a superposition of a membrane element and a plate element. The element has in-plane and out-of-plane stiffness, providing restriction along the six degrees of freedom. As shown in Fig. 1c, the element resists in-plane forces \( F_{11} \) and \( F_{22} \), in-plane shear \( V_{12} \), transverse shear forces \( V_{13} \) and \( V_{23} \), out-of-plane bending moments \( M_{11} \) and \( M_{22} \) and torsional out-of-plane moment \( M_{12} \).

There are various formulations with different levels of accuracy available for these types of finite elements, including displacement based and stress based elements. In all the analyses presented in this paper, slabs and walls are modeled using the finite element developed for NASA [3, 4]. The membrane part of the element includes rotational degrees of freedom (\( R_3 \)). The plate part is
based on Reissner-Midlein plate theory considering transverse shear (thick plate). As in most commercial structural software, in the software used in this article [5], stiffness modifiers can be assigned to each internal force component.

**Application Example**

In order to illustrate the effect of the type of element used to model slabs in the seismic analysis of buildings, analyses are presented for a 15-story bearing wall system, consisting of special reinforced concrete shear walls and solid concrete slabs (industrialized system). The structure is presented in Fig. 2. The wall thickness is 250 mm and slab thickness 120 mm. Story height is 2.70 m, and the concrete compressive strength for slabs and walls is $f'_c = 28$ MPa.

![Figure 2. Typical floor framing plan.](image)

**Gravity Loads**

In most commercial structural software the distribution of slab loads to supporting walls depends on the type of element used to model slabs. When slabs are modeled as membrane elements, a tributary method of triangles and trapezoids is usually used. Such distribution is attractive as it resembles that obtained in simplified yield-theory analysis or in sophisticated nonlinear analysis using multilayer plate elements. However, most implementations are limited and only work well in cases of regular geometry with panels supported on all sides. When slabs are modeled as shell elements, floor loads are applied directly to the elements representing the slab. Such a procedure is general, and the results are representative of the load distribution in elastic conditions, as long as the slab panels are properly discretized.

In this article, independently of the type of element used to model the slabs, the distribution of floor loads was done automatically using a general tributary area procedure, which properly handles cases of complex geometry, and cases of slabs panels supported on any number of sides. Fig. 3 shows the tributary area assigned to each individual wall element. The same areas were used to distribute dead floor loads and live floor loads. The superimposed dead load (DLi), additional to the slab weight considered in the analyses, was 1.80 kPa and the live load was 2.0 kPa. The same floor load distribution was used in all the cases considered.
Seismic loads

The building was assumed to be located in a seismic area, and the following seismic parameters according to ASCE7-10/16 were used: \( S_s = 0.94, S_1 = 0.45, I = 1, S = D, R = 5, SDC = D \). In order not to introduce additional variables, the same seismic loads were considered in all the analyses. Such loads correspond to the seismic force distribution determined by the equivalent lateral load procedure. For simplicity, accidental torsion was not considered in the analyses. The structure was assumed fixed on a rigid mat footing.

Story Drift Ratios

Figure 3. Distribution of slab loads to supporting walls.

Figure 4. Story drift ratios – Slabs modeled with Membrane elements.
The story drift ratios calculated when the floor slabs are modeled as membrane elements are presented in Figure 4. The maximum drift ratio in the X direction is 5.4% and in the Y direction is 1.2%. The drift in the X direction is excessive given that most of the walls are oriented in the Y direction, while only a few short walls are oriented in the X-direction. These few short walls do not provide sufficient rigidity to the structure in this direction. Almost identical results are obtained if a rigid diaphragm condition is enforced for all floors slabs.

The story drift ratios calculated when the floor slabs are modeled as Shell elements are presented in Figure 5. The drift ratios are drastically smaller. The maximum drift ratio in the X direction is 0.79% and in the Y direction is 0.6%. Despite the marked structural difference in the two directions, with the majority of the walls oriented in the Y direction, the maximum story drift ratios computed for the two directions are similar.

**Required Reinforcement for Walls**

The results of the design of the special shear walls according to ACI-318-11, limiting the vertical shear ratio to 4%, for slabs modeled as (a) membranes and (b) shells, are presented in Figure 6. When slabs are modeled as membranes, various walls in the lower 8 stories result with insufficient section. Additionally, numerous wall elements require special boundary elements. The total weight of required reinforcement (vertical plus horizontal) exceeds 1,500 kN. On the contrary, if slabs are modeled as shell elements, most of the wall elements require minimum reinforcement, with some elements requiring concentration of reinforcement at the ends, and only a few requiring special boundary elements. The total weight of required reinforcement (vertical plus horizontal) is reduced to only 700 kN.

**Comparison of Results and Validity of the Models**

As the application example illustrates, there is a major difference in the seismic analysis and design results when slabs are modeled as diaphragms, which only have in-plane stiffness, and
when modeled as shell elements, which include out-of-plane stiffness. Modeling slabs as shells, reduce maximum story drift ratios from 5.4% to 0.79% and the required reinforcement of structural walls from 1500 kN to 700 kN. This huge difference in the results is another reason why some design offices have used the last option on a large number of building projects, some already built and others under construction, justifying their choice by arguing it is a more efficient design.

A close examination on how, in the analysis with Shell elements, slabs contribute to resist inertial seismic forces by out-of-plane bending, the magnitude of the internal forces developed on such elements under the earthquake loading, along with the consideration of the general practice to design slabs for gravity loads only, shows that the above argument is not valid.

In fact, instead of leading to a more optimum design, modeling floor slabs as Shell elements results in several harmful effects on the seismic design of the building. First, it involves the contribution of slabs to the system that provides lateral stiffness and lateral resistance, reducing the computed story drift ratios drastically, by resisting significant stresses for which they are not designed. Second, it reduces the internal forces on the actual elements of the seismic force-resisting system, leading to an unsafe design of such elements. Third, it leads to an underestimation of the seismic foundation loads for the vertical elements of the seismic force-resisting system, as shown in Fig. 7.

Although slabs are flexural elements and modeling them with Shell elements improves the representation of their behavior under gravity loads, such modeling alters the representation of the seismic force-resisting system by involving elements, which are not designed for such purpose. It is not appropriate to include as part of the seismic force-resisting system elements that are designed only for gravity loads.
Vertical Stresses on Structural Walls

Fig. 7 shows for load case Earthquake X, the mid-plane vertical stress \((S_v)\) as a fraction of concrete strength \((f'c)\), for the group of wall elements connected to one of the 8 short walls oriented in the X direction [wall I(10-11)]. For slabs modeled as Membranes, only those groups of walls provide significant lateral stiffness, as the contribution of isolated wall elements oriented in the Y direction is minimal, since they act as independent cantilevers bending in the weak direction. Seismic forces on the short wall elements result in significant flexural stresses. The short walls transfer significant stresses to the transverse walls connected to them, oriented in the Y direction, which act as flanges (one in compression and the other one in tension). The maximum compressive stress is \(0.86 f'c\) and the maximum tensile stress is \(0.4 f'c\).

In the case of slabs modeled as Shell elements, the slab creates a frame effect that couples all the wall elements in the floor framing plan, reducing the magnitude of the stresses on the individual wall elements. The maximum compressive stress is reduced to \(0.26 f'c\) and the maximum tensile stress is reduced to \(0.15 f'c\).

![Figure 7. Mid-plane vertical stress ratio: (a) Slabs as Membranes (b) Slabs as Shells](image)

**Boundary elements and steel ratio on structural walls**

The significant difference in the magnitude of flexural stresses on the structural walls, depending
on how slabs are modeled, have an important effect on the amount of reinforcement and the size of the special boundary elements. In Fig. 8, the required steel ratio and the size of special boundary elements for the first-story structural walls are presented for the case of (a) slabs modeled as Membranes and (b) slabs modeled as Shells. In the case of slabs modeled as Membranes, the steel ratio and size of boundary elements are significantly larger, including cases of multiple wall segments which require confinement on their full lengths. Wall segments oriented in X direction have insufficient section (require steel ratios greater that the specified limit of 4%). Such elements couple the transverse elements connected to them, resulting in large steel ratios and boundary elements for these wall elements as well. The steel ratio for some of the isolated transverse wall elements, oriented in the Y direction, is the minimum, which shows their little contribution to resist seismic loads. In the case of slabs modeled as Shells, the required steel ratio and the size of boundary elements is significantly smaller. Even for this first story, several wall elements do not require special boundary elements, including the 4 walls oriented in the X direction, which only require concentration of vertical steel reinforcement at the ends.

Figure 8. Vertical steel ratio and boundary elements (a) Slabs as Membranes (b) Slabs as Shells.

**Bending Moments in Slabs Modeled as Shells**

Slabs are normally designed for bending moments due to gravity loads, ignoring bending moments due to earthquake loading. However, if slabs are modeled as shell elements, to take advantage of the huge reduction in story drift ratios and the required reinforcement on the vertical elements of the seismic force-resisting system, resulting from considering the contribution of slabs to resist seismic forces through out-of-plane bending, it would be necessary to ensure that the slabs can actually resist those bending moments and shear forces produced by the design earthquake.

Fig. 9a shows for the case of slabs modeled as shell elements, the bending moments in longitudinal slab strips due to Earthquake in X direction (EQX). And, Fig. 9b shows, for the same model, the bending moments due to gravity loads (self-weight plus superimposed dead load). The results displayed are for floor level 5, however, results are similar for the other floor level of the building. The bending moment scale is the same in both cases. The figure shows that the bending moments due to earthquake loading are significantly greater that those due to gravity loading. It becomes clear that given the relative magnitude of moments, modeling slabs as shell elements and then designing them just for gravity loads, ignoring seismic bending moments, results in an unsafe design.
It is pointed out that the issue is not just the magnitude of the bending moments but their nature. To safely rely on the significant reduction on both story drift ratios and required reinforcement, resulting from considering that slabs contribute to resist seismic forces through out-of-plane bending, it would be required to ensure that slabs can actually resist those cyclic bending moments of Fig. 9a, according to the special provisions for earthquake resistance design (Chapter 21 ACI-318-11, Chapter 18 ACI-318-14). The purpose of such special provisions is to ensure flexural elements may resist such cyclic loading without significant damage during the full duration of the earthquake. These provisions include, among others: minimum dimensions, b/h ratios, confinement requirements with closed ties, provisions for lap splices, and shear strength requirements. To comply with those special seismic provisions in the case of a 120 mm–thick slab element is not feasible. For this reason slabs are not considered part of the seismic force-resisting system, for buildings assigned to SDC D to F.

**Slabs Modeled as partial Shells**

The analysis and design results for slabs modeled with shell elements discussed above are for complete shells. That is with no stiffness modification factors. Fig. 10 shows the results of the computed story drift ratio for different values of the stiffness reduction factor for the plate component. The results show that as the stiffness reduction factor increases from 0 to 1.0, that is, as the slab contribution through out-of-plane bending increases, the difference in stiffness in
the two directions decreases. The results also show that even if only 10% of the out-of-plane stiffness is considered, the computed maximum story drift ratio is reduced almost to one half (5.4% to 3.18%). Even for such a low stiffness reduction factor, the bending moments on the slab due to seismic forces are comparable to those due to gravity loads.

<table>
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<th>Finite Element</th>
<th>Reduction</th>
<th>Drift X (%)</th>
<th>Drift Y (%)</th>
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Figure 10. Story drift ratio slabs modeled as Shell elements with reduced out-of-plane stiffness

**Conclusions**

Due to the fact that it is not feasible to reinforce concrete slabs complying with the special provisions for earthquake resistant design for flexural members, slabs are not considered part of the seismic force-resisting system of buildings assigned to seismic design category D to F. Therefore, for such buildings, floor slabs should be modeled considering only their action as diaphragms (membranes), which distribute the inertial forces of the floor to the vertical elements of the seismic force-resisting system, through in-plane stresses.

Modeling floor slabs as Shell elements, introduces two major adverse effects on the seismic design of the structure. First, it involves the contribution of slabs to the system that provides lateral strength and stiffness to the building, reducing drastically the computed story drift ratios, by resisting significant stresses for which they are not designed. Second, it reduces the computed internal forces on the actual elements of the seismic force-resisting system, leading to an unsafe design of such elements.

Although slabs are flexural elements and a model based on shell elements improve their representation under gravity loads, using such model in the lateral load analysis distorts the representation of the seismic force-resisting system by involving elements that cannot be designed for such purpose.

When using commercial software that allows modeling slabs, either as shell or membrane elements, if using shell elements, it is recommended to apply stiffness modification factors to the plate component of the slab element. A common mistake is to apply stiffness reduction factors to only one flexural component. Reduction factors should be applied to all the internal force
components of the plate element, including bending moments $M_{11}$ and $M_{22}$, torsion moment $M_{12}$, and transverse shears $V_{13}$ and $V_{23}$. It is recommended to use stiffness reduction factors less than 0.01.

Another option available in commercial software is to assign a reduced slab thickness for the plate component. Although such reduced thickness may distort the distribution of gravity loads, it properly reduces the out-of-plane stiffness contribution of the slab in the seismic analysis. It is recommended to use a thickness less than 0.2 times the real thickness of the slab. The reduced thickness should not be applied to the membrane component, as the diaphragm effect of the slab would be reduced, distorting the distribution of inertial forces.

In short, structural software packages are only computational tools that offer different options to model buildings. The way structural engineers achieve a safe design, which complies with objectives of the building code provisions, is through proper modeling and a design based on realistic assumptions. The use of full shell elements to model concrete slabs is an example of improper use of structural software.

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