RELIABILITY OF ENGINEERING SYSTEMS COMBINING STRUCTURAL HEALTH MONITORING WITH STATE-OF-THE-ART DETERIORATION MODELS

L. Iannaccone\(^1\) and P. Gardoni\(^2\)

ABSTRACT

The probability of failure of engineering systems is typically time variant due to the effects of environmental conditions and operational loads. Such effects affect the system properties (state variables) that define the system ability to sustain future demands and the demands they might be facing. Therefore, it is of primary importance to estimate values of the state variables over time. Structural Health Monitoring (SHM) and Non-Destructive Evaluation (NDE) can be used in estimating the state variables at different times. However, typically this identification process is an ill-defined problem, i.e. different combination of the state variables can be possible to achieve the same values from SHM or NDE. Also SHM and NDE values cannot be used directly to obtain estimates of the probability of failure of the system at future times. To address these issues, this paper couples SHM and NDE with physics-based probabilistic models of the state variables that capture the physics of the deterioration process. The models can be calibrated using SHM and NDE data in a well-defined problem and can be used to estimate the values of the state variables at future times.

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Reliability of Engineering Systems Combining Structural Health Monitoring with State-of-the-Art Deterioration Models

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ABSTRACT

The probability of failure of engineering systems is typically time variant due to the effects of environmental conditions and operational loads. Such effects affect the system properties (state variables) that define the system ability to sustain future demands and the demands they might be facing. Therefore, it is of primary importance to estimate values of the state variables over time. Structural Health Monitoring (SHM) and Non-Destructive Evaluation (NDE) can be used in estimating the state variables at different times. However, typically this identification process is an ill-defined problem, i.e. different combination of the state variables can be possible to achieve the same values from SHM or NDE. Also SHM and NDE values cannot be used directly to obtain estimates of the probability of failure of the system at future times. To address these issues, this paper couples SHM and NDE with physics-based probabilistic models of the state variables that capture the physics of the deterioration process. The models can be calibrated using SHM and NDE data in a well-defined problem and can be used to estimate the values of the state variables at future times.

Introduction

Engineering systems are subject to deterioration processes that might increase their probability of failure over time. In order to minimize disruption and device optimal maintenance and operation strategies, we want to predict the effect of these processes on their reliability. Deterioration processes can be divided into gradual (such as corrosion and fatigue) and shock (such as the effect of earthquakes). Models have been developed recently ([1-3]) that take into account the interactions between deterioration processes by using state-dependent models for the state variables of the system. In addition, Non-Destructive Evaluation (NDE) procedures and Structural Health Monitoring (SHM) can provide information to update the knowledge about the state variables of the system at the time of the inspection, but there is limited literature on how to use the data coming NDE/SHM to update the models for the deterioration processes acting on the system. By coupling the results coming from NDE/SHM with state-of-the-art deterioration models, we are able to accurately update our knowledge on how the system is deteriorating over time, which in turn allows us to estimate the reliability of the system at times following the time of inspection.

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Calibration of state-dependent models for deterioration using data from NDE and SHM

Let \( \mathbf{X}(t) \) be the vector of the state variables, i.e. the variables that affect the performance of the system, such as material properties or geometry. If we call \( \mathbf{X}_0 = \mathbf{X}(t = 0) \) the vector of state variables at the reference time \( t = 0 \), we can express the vector of the state variables at time \( t \) following [1-3] as \( \mathbf{X}(t) = \mathbf{X}_0 + \int_0^t \dot{\mathbf{X}}(\tau) d\tau \) where \( \dot{\mathbf{X}}(t) \) is the rate of state change at time \( t \). If \( m \) different deterioration processes are acting on the system, \( \dot{\mathbf{X}}(t) \) is given by

\[
\dot{\mathbf{X}}(t) = \sum_{k=1}^m \dot{\mathbf{X}}_k[t, \mathbf{X}(t), \mathbf{Z}_k(t); \Theta_{x,k}]
\]  

(1)

where \( \dot{\mathbf{X}}_k(t) \) denotes the rate of state change due to the \( k^{th} \) deterioration process, \( \mathbf{Z}_k(t) \) is the vector of the external variables that affect the \( k^{th} \) deterioration process, and \( \Theta_{x,k} \) is a vector of model parameters to be calibrated based on the data from NDE and SHM. By making the rate of change at time \( t \) a function of the state of the variables at time \( t \) itself, \( \dot{\mathbf{X}}(t) \) is able to capture the interaction between different deterioration processes. Assume now that \( n \) collections of field data from NDE/SHM \( \mathbf{x}_{Field} \) are available at \( n \) different moments in time \( \tau = [\tau_1, ..., \tau_n] \). This information can be used to obtain estimates of the model parameters \( \Theta_{x} = (\Theta_{x,1}, ..., \Theta_{x,m}) \). The parameters can be estimated using a Bayesian approach

\[
f(\Theta_{x} | \mathbf{x}_{Field}(\tau_j), j = 1, ..., n) = \kappa L(\mathbf{x}_{Field}(\tau_j), j = 1, ..., n | \Theta_{x}) p(\Theta_{x})
\]  

(2)

where \( f(\Theta_{x} | \mathbf{x}_{Field}(\tau_j), i = 1, ..., n) \) is the posterior distribution for the parameters, \( \kappa \) is a normalizing factor, \( L(\mathbf{x}_{Field}(\tau_j), i = 1, ..., n | \Theta_{x}) \) is the likelihood function proportional to the probability of observing the collected data given specific values for the parameters \( \Theta_{x} \), and \( p(\Theta_{x}) \) is the prior distribution reflecting the state of knowledge about \( \Theta_{x} \) prior to the collection of the data. The likelihood function can be written as

\[
L(\mathbf{x}_{Field}|\Theta_{x}) = \prod_{j=1}^n P[\mathbf{X}(\tau_j) = \mathbf{x}(\tau_j)|\mathbf{X}(\tau_{j-1}) = \mathbf{x}(\tau_{j-1}); \Theta_{x}]
\]  

(3)

The probabilities being multiplied in Eq. (3) are usually not available in closed form for state-dependent processes, but they can be obtained via Monte Carlo Simulation of subsequent realizations for the deterioration processes. We can generate \( n_s \) values of the state variable \( X_k \) at \( \tau_j \) (\( n_s \) being a sufficiently large number), considering that \( X_k(\tau_j) = x_k(\tau_{j-1}) + \sum_{i=1}^q \Delta X_k(t_i) \), where \( x_k(\tau_{j-1}) \) is the value of the \( k^{th} \) state variable from the \((j-1)^{th}\) NDE/SHM field data, and \( \Delta X_k(t_i) \) is the discrete change in \( X_k \) occurring in the \( i^{th} \) time interval between \( \tau_j \) and \( \tau_{j-1} \), which can be randomly generated using Eq.(1). The samples obtained for \( X_k(\tau_j) \) can then be fitted to a nonparametric PDF using Kernel Density Estimation (KDE) to obtain the conditional PDF \( f(X_k(\tau_j)|X_k(\tau_{j-1})) \). This PDF can be used to obtain the probability values in Eq. (3). Once the posterior distribution of the parameters \( \Theta_{x} \) has been obtained, the reliability of the system can be updated for the time of the inspection and any instant in the future.

Application to RC bridge columns – Lavic Road Bridge

As an application of the theory exposed in the previous section, we develop fragility estimates for
the Lavic Road Bridge in the San Bernardino County, California. Dimensions and important quantities for the bridge are available in Figure 1. Additional details can be found in [4].

Figure 1. Layout and measurements for Lavic Road bridge

Both gradual and shock processes are included in the model. The gradual deterioration process is considered to affect the diameter of the bar according to the following relationship (adapted from [5])

$$
\Delta d_b(t, T_{corr}) = \begin{cases} 
0 \\
- \frac{\theta_{d,1}(1-w/c)}{d_c} \theta_{d,3} [(t - T_{corr})^{\theta_{d,3}-1}] + \sigma_d \varepsilon_d 
\end{cases} \quad \text{for } t \leq T_{corr}
$$

$$
\Delta d_b(t, T_{corr}) = - \frac{\theta_{d,1}(1-w/c)}{d_c} \theta_{d,3} [(t - T_{corr})^{\theta_{d,3}-1}] + \sigma_d \varepsilon_d \quad \text{for } t > T_{corr}
$$

(4)

where $T_{corr}$ is the corrosion initiation time, $w/c$ is the water-to-cement ratio, $\theta_{d,1}$, $\theta_{d,2}$, $\theta_{d,3}$ and $\sigma_d$ are uncertain parameters. We assume that $\sigma_d$ is a constant standard deviation of the model error and $\varepsilon_d$ is a normally distributed random variable. Shock deterioration processes are assumed to affect both the lateral stiffness $K$ of the column and the displacement at yield $\Delta y$ [5]

$$
\ln[K(t^+)] = [\theta_{K,1} + \theta_{K,2} P_u/(f_c A_g) + \theta_{K,3} T_n(t^-)] \ln[\delta_D(t^-)] + \sigma_K \varepsilon_K
$$

(5)

$$
\ln[\Delta y(t^+)] = \theta_{\Delta,1} \ln[K(t^+)] + \sigma_{\Delta} \varepsilon_{\Delta}
$$

(6)

where $t^-$ and $t^+$ denote the instants right before and right after the occurrence of an earthquake, $f_c$ is the compressive strength of concrete, $A_g$ is the cross-sectional area of the column, $T_n$ is the fundamental period of the structure and $\delta_D$ is the maximum drift during an earthquake. $\theta_{K,1}$, $\theta_{K,2}$, $\theta_{K,3}$, $\theta_{\Delta,1}$, $\sigma_K$ and $\sigma_{\Delta}$ are uncertain model parameters. We assume that $\sigma_K$ and $\sigma_{\Delta}$ are constant standard deviation of the model errors and $\varepsilon_K$ and $\varepsilon_{\Delta}$ are correlated random variables with bivariate normal distribution and correlation coefficient $\rho_{K,\Delta}$. These parameters are calibrated based on the modal analysis performed at four different times, from which the evaluated stiffness is shown in Figure 2(a) (from [4]). Note that the stiffness computed after September 1998 had to be scaled due to different instrumentation being used ([4]) and that there is a drastic decrease in stiffness from September 1999 to October 1999. This is due to the occurrence of an earthquake of Magnitude 7.1 (the Hector Mine earthquake), with epicenter 16 km away from the location of the bridge. The procedure exposed in the previous section is used to calibrate the parameters in the deterioration models in Eq. (4-6). Figure 2(b) shows a comparison between the fragility curve for $t = 0$, $t = 33$ and $t = 75$ years. Solid lines show the predictive estimates and the bands represent the approximate confidence bounds [6]. As expected, higher values in the fragility imply a higher probability of failure as the bridge ages and deteriorates.
Figure 2. (a) Flexural stiffness EI resulting from NDE of the bridge column ($10^6$ kN m$^2$) and (b) Fragility curves for the considered bridge at different times

**Conclusions**

A procedure to include the result from Non-Destructive Evaluation and Structural Health Monitoring into the estimate of the fragility of engineering system and the calibration of the models for deterioration was provided. Results show how the developed method allows to update the knowledge about the state of the system at the time of the inspection and, in addition to most of the current procedures, to calibrate deterioration models to obtain estimates of the fragility at any moment after the latest inspection.

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**References**


