FORMULATION OF STRUT-AND-TIE MODEL FOR ESTIMATING THE SHEAR CAPACITY OF CONFINED MASONRY WALLS

D. Tripathy¹ and V. Singhal²

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Strut-and-tie model has evolved as the most useful method for the shear critical structures and disturbed regions (often called as D-region) in concrete structures and thus can be applicable to confined masonry (CM) walls, which often behave as shear wall. To estimate the in-plane capacity of CM wall using strut-and-tie methodology, a prior knowledge of limiting strength of struts and ties is required. The strength of ties can be simply determined from the yield strength of longitudinal reinforcement. However, the limiting capacity of the strut may depend on various parameters such as aspect ratio of wall, strength of masonry as well as concrete and cross-section detail of tie-column and tie-beam. Therefore, to estimate the strength of strut, a parametric study was performed considering various parameters through non-linear finite element (FE) analyses. In total 216 FE models of CM walls were developed to understand the behavior of strut under in-plane loads. Based on the FE study, semi-empirical equations were proposed to determine the limiting capacity of the strut. Subsequently, a repository of about 50 test specimens was created and the proposed equations were used to formulate the strut-and-tie model for these specimens. It has been observed that the developed strut-and-tie models consistently provided good predictions for the in-plane capacity of CM walls within an error of 20% when compared to the experimental results. Due to its simplicity and reliability, the strut-and-tie model can be put into practice for the analysis and design of single to multi-story confined masonry buildings.

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ABSTRACT

Strut-and-tie model has evolved as the most useful method for the shear critical structures and disturbed regions (often called as D-region) in concrete structures and thus can be applicable to confined masonry (CM) walls, which often behave as shear wall. To estimate the in-plane capacity of CM wall using strut-and-tie methodology, a prior knowledge of limiting strength of struts and ties is required. The strength of ties can be simply determined from the yield strength of longitudinal reinforcement. However, the limiting capacity of the strut may depend on various parameters such as aspect ratio of wall, strength of masonry as well as concrete and cross-section detail of tie-column and tie-beam. Therefore, to estimate the strength of strut, a parametric study was performed considering various parameters through non-linear finite element (FE) analyses. In total 216 FE models of CM walls were developed to understand the behavior of strut under in-plane loads. Based on the FE study, semi-empirical equations were proposed to determine the limiting capacity of the strut. Subsequently, a repository of about 50 test specimens was created and the proposed equations were used to formulate the strut-and-tie model for these specimens. It has been observed that the developed strut-and-tie models consistently provided good predictions for the in-plane capacity of CM walls within an error of 20% when compared to the experimental results. Due to its simplicity and reliability, the strut-and-tie model can be put into practice for the analysis and design of single to multi-story confined masonry buildings.

Introduction

The confined masonry (CM) structure consists of load bearing masonry walls strengthened with nominally reinforced concrete elements at the perimeter and other key locations. These vertical and horizontal reinforced concrete elements are popularly known as tie-column and tie-beam, respectively. The confined masonry system has evolved based on its satisfactory performance in past earthquakes and can be considered as one of the most suitable alternatives to seismically vulnerable unreinforced masonry system due to its similar construction practice and economic

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feasibility. However, the existing analytical models for estimating the design shear capacity of CM walls are primarily semi-empirical equations which are based on either friction theory or elementary theory of elasticity. These analytical equations are either highly influenced by the formulae originally developed for unreinforced and reinforced masonry walls or are based on the limited number of laboratory tests. The inconsistent predictions of in-plane shear strength by these existing models necessitate the development of a rational design method for CM walls [1].

Strut-and-tie model has evolved as the most useful method for shear critical structures and disturbed regions (often called as D-region) in concrete structures. This method provides a rational and consistent design approach by idealizing the structural member with appropriate truss models. However, a prior knowledge of flow of stresses is required to develop strut-and-tie model for complex structural members, such as CM walls with multiple panels or with openings. Thus, parametric nonlinear finite element analyses were performed on CM walls, which assisted in determining the stress flow and subsequently in developing the relationship between strut capacity and relative strength/stiffness of tie-elements. This paper will first briefly discuss the parametric finite element analyses of CM walls and lastly the proposed methodology for developing the strut-and-tie model for estimating the in-plane strength of CM walls.

Application of Strut-and-Tie Method to Confined Masonry Walls

Confined masonry walls often behave as shear critical structures such as shear walls. Therefore, the strut-and-tie method can be a possible alternative for predicting the in-plane shear capacity of CM wall due to its considerable positive outcomes in case of distributed or shear critical regions. Fig. 1a represents a CM wall, which is subjected to moment (M) and axial load (P) [2]. The tensile stresses as shown in Fig. 1a are taken by longitudinal reinforcement in tie-columns, whereas the compressive stresses are resisted by concrete and masonry. Hence, similar to RC wall, the strut-and-tie model (STM) for CM wall will consist of pin-jointed structural truss which connects both tension and compression members. A possible STM for the CM walls based on its load resisting mechanism is represented in Fig. 1b. The broken and solid lines in Fig. 1b represent the strut and tie, respectively.

Figure 1. (a) Internal force distribution in a CM wall subjected to axial load and bending (redrawn from Meli et al. [2]), and (b) a possible strut-and-tie model for CM wall
To develop strut-and-tie model for a CM wall, the basic design requirements are as follows [3];

a) masonry should have sufficient shear resistance to carry diagonal strut forces,

b) horizontal and vertical strut forces should be resisted by concrete members, i.e. confining elements, and

c) amount of reinforcement in confining elements should be computed from tie forces.

Use of strut-and-tie method to CM wall is still in infancy. Recently Ghaisas et al. [4] have suggested the guidelines for forming the STM for CM wall through linear FE analysis. However, no details are provided on how to estimate the design in-plane shear capacity of CM walls. To estimate the in-plane capacity of CM wall using strut-and-tie method, limiting capacity of strut-and-tie should be known. The limiting strength of tie can be determined using the tensile strength of longitudinal reinforcement. However, the limiting capacity of the strut may depend on the various parameters such as the aspect ratio of wall, the relative stiffness of masonry wall and confining elements and the material properties. Therefore, to incorporate the effect of all these factors, a parametric study was performed.

**Parametric Finite Element Study**

A parametric study has been performed considering various properties like the geometry of wall panel, material characteristic and cross-sectional details of confining elements. Six aspect ratios have been considered in the following study, viz. 0.5, 0.75, 1.0, 1.5, 1.75, and 2.13. Height of the wall was kept as 2.5 m in all case. Ranges of various parameter considered in this study are listed in Table 1. In total 216 FE models were developed. Figs. 2a-2f illustrate samples of these FE models developed for each aspect ratio.

**Table 1. Ranges of design variables for the simulation of CM wall**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength of masonry ($f_{cm}$)</td>
<td>2.9, 5.9 and 8.5 MPa</td>
</tr>
<tr>
<td>Modulus of elasticity of masonry ($E_{cm}$)</td>
<td>1795, 3509 and 5219 MPa</td>
</tr>
<tr>
<td>Compressive strength of concrete ($f_{cc}$)</td>
<td>20, 25 and 30 MPa</td>
</tr>
<tr>
<td>Modulus of elasticity of concrete ($E_{c}$)</td>
<td>22360, 25000 and 27386 MPa</td>
</tr>
<tr>
<td>Aspect ratio [AR($=H_w/L_w$)]</td>
<td>0.5, 0.75, 1, 1.5, 1.75 and 2.13</td>
</tr>
<tr>
<td>Total height of wall ($H_t$)</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Thickness of wall ($t_w$)</td>
<td>0.12 and 0.24 m</td>
</tr>
<tr>
<td>Depth of tie-beam ($w_{tb}$)</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Width of tie-beam ($t_{tb}$)</td>
<td>0.12 and 0.24 m</td>
</tr>
<tr>
<td>Depth of tie-column ($w_{tc}$)</td>
<td>0.15, 0.2, 0.25 and 0.3 m</td>
</tr>
<tr>
<td>Width of tie-column ($t_{tc}$)</td>
<td>0.12 and 0.24 m</td>
</tr>
<tr>
<td>Longitudinal reinforcement ratio in confining elements ($\rho_{lc}, \rho_{tb}$)</td>
<td>1.7%</td>
</tr>
</tbody>
</table>

**Finite Element Modeling**

To conduct the parametric study, macro-modeling technique is used for developing the FE models using Abaqus [5]. In this study, the masonry was modeled at the macroscopic level while
the interaction properties and other confining elements were made at microscopic level. The 8-noded 3D element with reduced integration (C3D8R) was used to model the masonry panel and confining elements. However, 2-noded 3D truss elements (T3D2) were used to model rebars, which were embedded in the surrounding concrete elements. Tie constraints were provided to model the interaction behavior at all wall-to-tie-element interfaces except at the top surface of bottom tie-member on which the masonry was laid. To account for the slip phenomena at such interfaces, appropriate interaction was specified in both tangential (friction factor, $\mu_f = 0.6$) and normal direction (hard contact). The appropriate boundary conditions in this study were achieved by restraining all the nodes at the base to simulate a fixed boundary condition. The developed FE models were subjected to gravity loading followed by constant vertical loading and lateral displacement. An overburden pressure of 0.1 MPa and lateral displacement of 25 mm were considered to carry out the simulation of developed models. Abaqus/Explicit technique was used in this study based on its suitability in capturing the non-linear behavior of masonry wall whereas Concrete Damaged Plasticity (CDP) model was used to simulate the inelastic behavior of masonry and concrete. The developed FE models were calibrated and validated using the experimental study conducted by Gavilan et al. [6].

**Material model**

The constitutive model for masonry in compression was developed using the simplified tri-linear curve as shown in Fig. 3a [7]. The tensile behavior of masonry was simulated using a bi-linear curve (see Fig. 3b). The ascending branch of this curve illustrates the elastic behavior of masonry whereas the descending linear branch represents the post–yield behavior. The tensile strength of masonry $f_{mt}$, was taken as one-tenth of its compressive strength [8]. Equations representing both the curves are given in Figs. 3a and 3b.

The model proposed by Kent and Park [9] was specified to describe the compressive behavior of concrete as shown in Fig. 4a. The tensile stress-strain behavior of concrete was defined based on the tension softening law proposed by Gopalaratnam and Shah [10] (see Fig. 4b). Strains corresponding to peak compressive stress, $\varepsilon_{cc}$ and tension stress, $\varepsilon_{ct}$ were taken as 0.002 and 0.00015, respectively. To define the post-peak behavior of concrete in tension region, various parameters such as crack width ($w_c$) and constants like $k_{ct}$, $\lambda_{ct}$ are required to be specified. In this study, the value of $k_{ct}$ and $\lambda_{ct}$ were considered as 0.063 and 1.01 respectively, while the value of crack width ($w_c$) was expressed in micrometers [11]. Other required material properties were taken as default values in Abaqus [5].
Figure 3. Material model of masonry considered for numerical modeling: a) compressive stress-strain curve, and b) tensile stress-strain curve

Result of Parametric FE study

The stress vector plots for the CM wall with aspect ratio ranging from 0.5-2.13 are represented in Fig. 5. As depicted from these figure, the stresses are transferred diagonally and strut width gradually increases with decrease in the aspect ratio. The influence of masonry and concrete strength and cross-section details of tie-column on the behavior of CM walls is shown through load-displacement curves in Figs. 6a, 6b and 6c. The plots shown in these figures are developed for aspect ratio 1.0, however, similar trend was observed for all other aspect ratios. Fig. 6a shows that masonry compressive strength, \( f_{mc} \) significantly influence the shear strength of CM wall.
As expected, the in-plane shear capacity increases with increase in masonry compression strength. However, the strength of concrete in the tie-column and size of tie-elements do not significantly affect the shear capacity of CM walls. To normalize the effect of these properties, a factor relative stiffness has been introduced. The value of relative stiffness in this study was estimated using Eq. 1, which is also popularly used for RC infill masonry frame [12].

\[
\lambda = H_T \left( \frac{E_m t_w \sin 2\theta}{4E_c I_c H_T} \right)^{0.25}
\]

where \(E_m\), \(t_w\), \(E_c\), \(I_c\) and \(H_T\) are variables in Eq. 1, which signifies modulus of masonry, wall thickness, modulus of concrete, moment of inertia of tie-column, and total height of wall including the width of tie-beam, respectively.
From the parametric study, generally, it has been found that the yielding of longitudinal bars started after reaching the peak lateral load of CM walls. This implies that the in-plane capacity of CM wall corresponds to strength of the masonry strut. Thus, the strut capacity $F_s$ is calculated by considering the cosine component of maximum lateral load obtained from FE analyses. Figs. 7a-7f show the relationship between strut strength $f_s$ and relative stiffness $\lambda$ for all considered aspect ratios. Strut strength is calculated by dividing strut capacity with the area of panel. Figs. 7a-7f illustrate that the strength of strut reduces with increase in relative stiffness. Using these figures, strut force can be calculated for a given aspect ratios, strength of masonry and geometric properties of the panel.

![Figure 7](image_url)

Figure 7. Effect of relative stiffness on the strut strength of the CM wall having aspect ratio (a) 2.13, (b) 1.75, (c) 1.5, (d) 1.0, (e) 0.75, and (f) 0.5

To propose a simplified generalized equation, the effect of various parameters such as concrete strength in tie-elements, geometric characteristic of wall panel as well as tie-elements and strength of masonry is normalized. Besides, the relative stiffness is sensitive to aspect ratio. Hence, it is normalized corresponded to different aspect ratios. The normalized relative stiffness is plotted with normalized strut strength to obtain a semi-empirical relation. As a result, two equations (Eq. 2 and Eq. 3) are developed for two different ranges of aspect ratio, $AR \leq 1.3$ and $AR > 1.3$ (see Figs. 8a and 8b).

$$F_s = 0.18 f_{mc} A_p \lambda \left(\frac{1}{AR} \right)^{1/4}$$ for $AR > 1.3$  \hspace{1cm} (2)
\[ F_s = 0.12 f_{mc} A_p \lambda^{(AR/40)^2} \] for \( AR \leq 1.3 \) \hfill (3)

where \( A_p \) is the area of panel excluding the width of tie-beam and tie-column.

Figure 8. Effect of normalized relative stiffness on the strut strength of CM walls with aspect ratio (a) greater than 1.3, (b) less than equal to 1.3

**In-plane strength prediction using proposed equation**

To evaluate the accuracy of proposed equations in predicting the in-plane shear capacity of CM walls, a repository of 50 test specimens available in literature was created. This repository consists of solid CM wall specimens covering a wide range of material properties, aspect ratio, cross-sectional details and number of masonry panels. Ranges of various parameters in the developed repository are listed in Table 2.

**Table 2. Ranges of Design Variables for CM wall**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial shear strength without compression load ( (v_{\text{mu}}) )</td>
<td>0.3 – 1.1 MPa</td>
</tr>
<tr>
<td>Compressive strength of masonry ( (f_{mc}) )</td>
<td>2.66 – 35 MPa</td>
</tr>
<tr>
<td>Pre-compression force ( (\sigma_o) )</td>
<td>0 – 1.36 MPa</td>
</tr>
<tr>
<td>Compressive strength of concrete ( (f_c) )</td>
<td>20 – 35 MPa</td>
</tr>
<tr>
<td>Yielding strength of longitudinal bars ( (f_y) )</td>
<td>330 – 450 MPa</td>
</tr>
<tr>
<td>Joint compressive strength ( (f_j) )</td>
<td>( \leq 25 ) MPa</td>
</tr>
<tr>
<td>Longitudinal reinforcement ratio for tie-column ( (\rho_{lc}) )</td>
<td>( \leq 0.03 )</td>
</tr>
<tr>
<td>No of tie-columns ( (N_{tc}) )</td>
<td>2 – 4</td>
</tr>
<tr>
<td>Aspect ratio ( (H_w/L_w) )</td>
<td>0.27 – 2.13</td>
</tr>
<tr>
<td>Thickness of wall ( (t_w) )</td>
<td>60 – 200 mm</td>
</tr>
</tbody>
</table>
Proposed Strut-and-Tie Analysis for Solid CM Wall

At first, strut-and-tie analysis was performed by assuming the yielding of longitudinal reinforcement in tie-columns. The force in ties was estimated using the yield strength of longitudinal bars and by maintaining equilibrium at each node, strut force was determined. If the force in strut exceeds its capacity, then it was set to limiting value obtained using the proposed equations (2) and (3). Accordingly, the analysis was revised and the lateral load of CM walls was determined. Figs. 9a and 9b show strut-and-tie analysis for a single and double paneled CM walls chosen from the repository. The broken and solid line in Fig. 9 indicates strut and tie component, respectively. The estimated lateral load as shown in Fig 9a and 9b match closely with their respective experimental result.

![Figure 9. Strut-and-tie model: (a) single panel CM wall (b) multi-panel CM wall](image)

The strength predicted using the proposed strut-and-tie method was compared with their respective experimental result. The result suggests that the strut-and-tie analysis performed using the proposed equations consistently provided good predictions for the in-plane capacity of CM walls within an error of about 20% as shown in Fig. 10.

![Figure 10. Error in prediction of in-plane capacity using proposed strut-and-tie analysis](image)
Conclusion

Application of strut-and-tie method for estimating the capacity of CM walls has been described in this paper. To perform the strut-and-tie analysis, a prior knowledge of limiting capacity of strut and tie should be known. To estimate strut capacity, a non-linear parametric study was performed on the 216 FE models considering a wide range of various properties such as masonry and concrete strength, size of tie-column and aspect ratios. Using the FE results, two equations were proposed for two different ranges of aspect ratios, AR ≤ 1.3 and AR > 1.3. These equations were used to develop the strut-and-tie model and thus to calculate the in-plane shear capacity of various CM specimens chosen from experimental studies available in the literature. It has been observed that the proposed methodology consistently provided good predictions for the in-plane capacity of CM walls within an error of 20% when compared to experimental results. This study is limited to solid CM wall and can be further extended for estimating the shear capacity of CM walls with openings.

Acknowledgments

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References