RELIABILITY ANALYSIS OF A SHEAR-CRITICAL BEAM

O. Tunaboyu\textsuperscript{1}, Ö. Yurdakul\textsuperscript{2}, Ö. Korkmaz\textsuperscript{3}, L. Routil\textsuperscript{4} and Ö. Avşar\textsuperscript{5}

ABSTRACT

The response of a reinforced concrete beam constructed without transverse reinforcement to achieve shear failure was investigated by experimental and numerical methods. Due to inherent uncertainties in material constitutive models, a nonlinear finite element method (FEM) was combined with a suitable stochastic sampling technique to propose a more advanced model for estimating the response of a shear-critical beam. For this purpose, the specimen was first tested under monotonic loading up to shear failure by a four-point bending test. Then, the stochastic model was developed by using Latin Hypercube Sampling (LHS) including statistical correlation among the prominent material parameters. Random parameters of concrete and reinforcement steel were defined in accordance with the material test results and code recommendations. The constituent outcomes of the stochastic model including a set of load-displacement curves are presented. The results of the stochastic approach matched well with the behavior of the specimen observed during the experimental test. The probability density function for ultimate load was obtained. After that, the reliability of the member for the ultimate limit state was compared with the code requirements to ensure the safe loading range. The design load, which corresponds the failure probability related to ultimate limit state was computed. Moreover, a simplified ECOV (Estimation of Coefficient of Variation) method was carried out to estimate the design load. It is found that the load obtained from reliability analyses for design load was reasonably in good agreement with the code recommended value.

\textsuperscript{1} Ph.D., Dept. of Civil Eng., Anadolu University, Eskişehir, Turkey, TR 26170 (onurtunaboyu@anadolu.edu.tr)
\textsuperscript{2} Ph.D. Candidate, Dept. of Transport Structures, University of Pardubice, Pardubice, Czech Republic, CZ 53210
\textsuperscript{3} Civil Eng., ASF-ABT Joint Venture, Istanbul, Turkey
\textsuperscript{4} Assist. Prof., Dept. of Transport Structures, University of Pardubice, Pardubice, Czech Republic, CZ 53210
\textsuperscript{5} Assoc. Prof., Dept. of Civil Eng., Anadolu University, Eskişehir, Turkey, TR 26170

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The response of a reinforced concrete beam constructed without transverse reinforcement to achieve shear failure was investigated by experimental and numerical methods. Due to inherent uncertainties in material constitutive models, a nonlinear finite element method (FEM) was combined with a suitable stochastic sampling technique to propose a more advanced model for estimating the response of a shear-critical beam. For this purpose, the specimen was first tested under monotonic loading up to shear failure by a four-point bending test. Then, the stochastic model was developed by using Latin Hypercube Sampling (LHS) including statistical correlation among the prominent material parameters. Random parameters of concrete and reinforcement steel were defined in accordance with the material test results and code recommendations. The constituent outcomes of the stochastic model including a set of load-displacement curves are presented. The results of the stochastic approach matched well with the behavior of the specimen observed during the experimental test. The probability density function for ultimate load was obtained. After that, the reliability of the member for the ultimate limit state was compared with the code requirements to ensure the safe loading range. The design load, which corresponds the failure probability related to ultimate limit state was computed. Moreover, a simplified ECOV (Estimation of Coefficient of Variation) method was carried out to estimate the design load. It is found that the load obtained from reliability analyses for design load was reasonably in good agreement with the code recommended value.

Introduction

A ductile response is expected from structural members according to capacity design principles. The requirements for such a failure mode and the design principles are described in several codes and guidelines. However, a considerable amount of existing reinforced concrete (RC) buildings in developing countries have specific deficiencies at local level, which can result in brittle failure of the member. The resulting local damages can also actuate the global failure mechanism, which brought the requirement to investigate the behavior of substandard members. Beam members as one of the important structural components in RC buildings with certain deficiencies were the subject of many studies in the literature. Many experimental and numerical studies have been performed to investigate the response of shear critical beams \cite{1-3}. This study also deals with the shear critical beam by considering inherent uncertainties in the structural components. Therefore, a suitable stochastic sampling technique (i.e. Latin hypercube sampling (LHS)) was combined with the nonlinear finite element method. The concrete and reinforcing steel were defined as random

\textsuperscript{1} Ph.D., Dept. of Civil Eng., Anadolu University, Eskişehir, Turkey, TR 26170 (onurtunaboyu@anadolu.edu.tr)
\textsuperscript{2} Ph.D. Candidate, Dept. of Transport Structures, University of Pardubice, Pardubice, Czech Republic, CZ 53210
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variables. The statistical correlation among the prominent material parameters was defined by a stochastic optimization technique, i.e. simulated annealing method [4]. The results from full probabilistic approach were presented in a form of statistical characteristics. Moreover, a simplified ECOV method was also implemented to estimate the design capacity.

**Experimental Study**

The experimental program consists of four-point bending test of a ½ scaled RC beam specimen. A gradually increasing displacement was applied up to shear failure of the beam. It was designed without any transverse reinforcement to results in beam shear failure due to exceeding the tensile capacity of concrete. The details such as dimension and reinforcement scheme of the specimen were presented in Figure 1.

![Figure 1. Reinforcement scheme and dimension details of the beam specimen](image)

**Numerical Study**

The modelling approach and its parameters such as material and section properties were created in accordance with the tested specimen in order to simulate the actual response. For modelling concrete geometry, a hexahedral element (i.e. CC IsoBrick) was used. CC3D NonLinCementitious2, which is a fracture-plastic concrete model, is used to combine constitutive models for tensile (fracturing) and compressive (plastic) behavior [5].

The compressive strength of the concrete, $f_c$, was obtained by material test results. The progress of the Menetrey and Willam [6] failure surface was controlled by the parameter $c \in (0,1)$ which determines the evolution of yield or crushing process. It is based on hardening/softening variable $f_c(\varepsilon_{eq}^p)$ [5]. This variable is obtained from the uniaxial stress-strain diagram of the concrete samples. Thus, an elliptical hardening and linear softening behavior were defined according to Van Mier [7] experimental observations as shown in Figure 2a and b. [5]. While the hardening part is strain-based, plastic displacement, $w_d$, is taken into account in the linear softening part. The relation between deformation (i.e. plastic displacement in compression, $w_d$, and crack opening in tension $w_c$), $w$, and strain, $\varepsilon$, was obtained by crack band theory, i.e. $w = \varepsilon L_t$ [8]. It should be noted that $L_t$ is the crack band size, which is derived from finite element size in case of finite element applications. A relation in a form of Gauss's function was adopted for reduction of compressive strength in the cracked concrete. The parameters were derived from the experimental data published by Kollegger and Mehlhorn [9], which included also Vecchio and Collins [10] test data [5]. The strength reduction factor, $c$, in Figure 2c was defined according to Dyngeland 1989 [11].
Figure 2. (a) Concrete in compression (b) Compression softening (b) Strength reduction in the cracked concrete [5]

The tensile behavior of plain concrete was assumed to be uncracked in the elastic region. The exponential softening relation within the stress in the crack cohesion, $\sigma$, and crack width, $w$, was defined according to Hordijk [12] in the post-elastic region (Figure 3a). $w_c$, which is the crack opening at full stress release, can be obtained by using $G_f - w$ relationship. Fracture energy of concrete, $G_f$, which is the required energy to generate the unit area of the crack surface, is the area under Stress–Crack Width curve. This value was computed as proposed by CEB-FIP Model Code 2010 [13], which equals to $73f_{ct}^{0.18}$ in N/m. The smeared crack concept in a combination of crack band theory by Bazant and Oh [8] was used for fracture model. For smeared manner, a fixed crack model by Cervenka [14] and Darvin and Pecknold [15] is adopted in the software, which employs a fixed crack direction after first cracking. Therefore, the axis of the orthotropy and axis of principal strain do not overlap, which creates shear crack on the crack face. Moreover, Rankine criterion is used for concrete cracking.

A reduction in the shear stiffness due to cracking of concrete was employed according to Kolmar [16]. By the increasing strain, which is normal to crack, the shear modulus is reduced; moreover, the dependence of reduction in the shear stiffness to transverse reinforcement ratio is also taken into account (Figure 3b).

Meshing can play a vital role in accuracy and reliability of the model since its efficacy is generally strongly correlated to mesh optimization. Different mesh sizes with same properties in all parts had been tried until the variation in the computed maximum load was minimized. A linear-hexahedral element with a size of 25 mm was employed to all parts as the final element size. The reinforcement steel bars were described by a bilinear model with hardening behavior.

Figure 3. (a) Tension softening (b) Shear retention factor [5]
Stochastic Study

Full Probabilistic Method

Due to inherent uncertainty in material constitutive models, the nonlinear finite element analysis was combined with a suitable stochastic sampling technique to propose an advanced tool for realistic assessment of the response of the shear critical beam. The prominent material inputs were defined as random variables. Those were obtained from either material test results or code recommendations. A stratified sampling technique, LHS, was employed to generate a random sample from the given distribution functions of the random parameters [17]. The statistical correlation among the prominent material parameters was defined by a stochastic optimization technique in FReET software, which is the simulated annealing method [4,17]. The randomized values and their distribution presented in Table 1 and correlation among the material parameters presented in Table 2 are based on Pukl et. al [17], fib Bulletin No.22 [18] and Joint Committee on Structural Safety (JCSS) [19]. A total of 30 simulations, which were the input parameters of the nonlinear FE solutions, were conducted in FReET software [4]. The number of simulations was determined in such a way that it was increased until there was no significant change in the computed parameters (e.g. correlation coefficients).

The distribution of the capacity can be obtained from the bundle of the stochastic model. For this purpose, the peak load values are firstly obtained from the set of load-displacement curves for each analysis. Then, their distribution function is obtained. The concept of reliability index was used to estimate the safety of residential buildings (RC2 type) where a medium consequence for loss of human life, economic, social or environmental consequences considerable is expected [20]. The Cornell reliability index and corresponding failure probability related to the ultimate limit state for the period of 50 years in RC2 types were 3.8 and 7.23E-05 [20]. The load corresponds the target probability is specified as the design value (capacity).

ECOV Method

ECOV (Estimation of Coefficient of Variation) method proposed by Cervenka [21] was implemented to evaluate the COV, \( V_r \), by using random distributions of mean, \( R_m \), and characteristic, \( R_k \), values of resistance (Eq. 1). A 2-parameter Lognormal distribution for design capacity, which could be approximated for most of the structures, is the main assumption of this method. Note that, EN 1990 [20] also recommends Lognormal distribution for the distribution of design capacity.

\[
V_r = \frac{1}{1.65} \ln \left( \frac{R_m}{R_k} \right) \tag{1}
\]

In general, the design value of resistance, \( R_d \), can be evaluated by Eq. 2 as proposed by CEB-FIP Model Code [13].

\[
R_d = \frac{R_m}{\gamma_r} \tag{2}
\]

In case of Lognormal distribution, the global safety factor, \( \gamma_r \), can be evaluated by Eq. 3 according to EN 1990 [20].
\[ \gamma_r = e^{\alpha_r \beta V_r} \]  
(3)

While sensitivity factor, \( \alpha_r \), is 0.8, Cornell reliability index, \( \beta \), is 3.8 as mentioned above. To obtain Coefficient of Variation, \( V_r \), two analyses are required. These analyses were performed with mean and characteristic values of basic material parameters. This clearly results in less computing time, which could be the main advantage of this method. It should be also noted that, while mean values were obtained from either material tests or codes, characteristic values were taken as 5% of mean values.

Table 1. Material properties as random parameters and their statistical distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean Value, ( \mu )</th>
<th>COV*</th>
<th>Distribution*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concrete</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, ( E_c ) (MPa)</td>
<td>4700, ( \sqrt{f_{c}}, ) [22]</td>
<td>0.10</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Tensile strength, ( f_{\text{st}} ) (MPa)</td>
<td>0.30( f_{c}^{2/3} ) [13]</td>
<td>0.30</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Compressive Strength, ( f_{c} ) (MPa)</td>
<td>15.0</td>
<td>0.15</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Fracture Energy, ( G_f ) (N/m)</td>
<td>0.000025( f_{\text{st}} ) [23]</td>
<td>0.25</td>
<td>Weibull (2 Parameter)</td>
</tr>
<tr>
<td>Compressive Strain, ( \varepsilon_{co} ) (mm/mm)</td>
<td>( f_{c}/E ) [7]</td>
<td>0.15</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Plastic Displacement, ( w_{d} ) (m)</td>
<td>0.0005 [7]</td>
<td>0.10</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td><strong>Reinforcing steel</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, ( E_s ) (GPa)</td>
<td>200</td>
<td>0.07</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Yield Strength, ( f_{y} ) (MPa)</td>
<td>460</td>
<td>0.07</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Ultimate Strength, ( f_{u} ) (MPa)</td>
<td>632</td>
<td>0.07</td>
<td>Lognormal (2 Parameter)</td>
</tr>
<tr>
<td>Ultimate Strain, ( \varepsilon_{u} ) (mm/mm)</td>
<td>0.05</td>
<td>0.07</td>
<td>Normal</td>
</tr>
</tbody>
</table>

* COV values and distribution functions are based on Pukl et. al [17], fib Bulletin No.22 [18] and JCSS [19]

Table 2. Correlation coefficients among the random parameters [17–19]

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_c )</td>
<td>( E_s )</td>
</tr>
<tr>
<td>( E_c )</td>
<td>1</td>
</tr>
<tr>
<td>( f_{c} )</td>
<td>1</td>
</tr>
<tr>
<td>( f_{st} )</td>
<td>1</td>
</tr>
<tr>
<td>( G_f )</td>
<td>1</td>
</tr>
<tr>
<td>( \varepsilon_{co} )</td>
<td></td>
</tr>
</tbody>
</table>

Results

The specimen underwent a sudden failure with the concentration of shear cracks on the left side of
the beam (Figure 4a). The overall response of the specimen had been dominated by beam yielding until shear response became critical. The beam could attain its flexural capacity (71.8 kN) up to certain level of displacement. Then, a distinct strength deterioration was observed as the beam reaches its shear capacity. Severe damage due to concrete cracking was observed in the subsequent displacement in the experiment on one side of the specimen. The crack pattern in the FE analysis distributed almost evenly to both sides of the specimens with more crack localization in one side of the beam (Figure 4b). The usage of the system at level of random fields results in random crack initiation due to the random distribution of concrete strength [24]. Therefore, this inconsistency in the crack localization in one side of the specimen can be attributed to the random distribution of concrete strength. In terms of capacity, the deterministic FE model, which was modelled with the average values of the material parameters, matched well with the experimental result in terms of capacity with an error less than 10% (Figure 5a and b). In the numerical models, the slope before peak load is steeper compared with the experiment. It results in a higher capacity in the initial slope while it well captures failure mode, crack pattern and capacity. Moreover, different kinds of failure modes such as beam yielding, concrete crushing were observed in the stochastic analysis.

Figure 4. Failed specimen (a) experiment (b) deterministic numerical model

Figure 5. (a) Set of load-displacement curves (b) Load-displacement curve with error bars

The probability density function (PDF) is the distribution of ultimate load values monitored in each analysis (Figure 6). The load-resistance interface method was used to obtain the safety or reliability margin, which can be computed by subtracting capacity (i.e. resistance) from demand (i.e. design load). As the target reliability index is known, the design load can be obtained by the reverse calculation.
The tail sensitivity is a significant parameter for estimation of design capacity as the corresponding probability for ultimate limit state takes place in the tail of probability density function (PDF). Therefore, accessible best fit distribution for PDF should be selected for more robust design. When the best fit curve is employed for the distribution of safety margin (i.e. Gumbel Min.), the obtained design force is 14.45 kN. In case of generally accepted distribution such as Lognormal as suggested by EN 1990 for design capacity [20] or Normal, a dramatic difference was found, which clearly shows the effect of the effect of PDF type to design load. Moreover, the design capacity for an unreinforced member in shear was also computed according to EN 1992-1-1 [25]. The obtained ultimate load and design load, $R_d$, were summarized in Table 3. When the results investigated deeply, full probabilistic approach with best-fit curve underestimated the capacity obtained from EN 1992-1-1 [25]. However, in case of code recommend distribution i.e. Lognormal distribution, design capacity computed by reliability analysis matched well with analytically obtained capacity. One can conclude that the tail sensitivity has a vital importance on the capacity.

Table 3. Capacity of the shear beam

<table>
<thead>
<tr>
<th></th>
<th>Ultimate Load (kN)</th>
<th>Design Capacity (kN)</th>
<th>$R_d$/EN 1992-1-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>71.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deterministic</td>
<td>74.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Probabilistic, PDF mean</td>
<td>70.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Probabilistic Design, $R_{dFL}$, Best fit curve, Gumbel Min</td>
<td>14.45</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>Full Probabilistic Design, $R_{dFL}$, Normal Distribution</td>
<td>40.00</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>Full Probabilistic Design, $R_{dFL}$, Lognormal Distribution</td>
<td>45.50</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>ECOV, $R_{dECOV}$</td>
<td>39.69</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>EN 1992-1-1 [25]</td>
<td>43.04</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

This study sets out the investigation of the response of a shear critical beam by experimental and numerical methods. Due to inherent uncertainties in the construction materials, the deterministic numerical model was evolved to the stochastic level. Therefore, LHS was combined with the nonlinear finite element method including statistical correlation among material parameters. Then, a set of load-displacement curves was found. To estimate the design capacity, two different stochastic approaches, which are full probabilistic method and ECOV method were used. Based on the results obtained in this study, the following conclusions can be drawn.

- The specimen exhibited a very sudden failure as a result of severe damage due to shear cracks.
- The ultimate load was determined as 71.31 kN and 74.74 kN in the experiment and deterministic model, respectively. It is obtained as 70.64 kN with the full probabilistic approach, which yields more accurate results.
- The value obtained from full probabilistic approach with best-fit curve underestimated the capacity when it was compared with EN 1992-1-1 [25].
- In case of generally accepted distributions such as Normal or Lognormal distributions for design capacity, the results from reliability analyses matched well with the analytically obtained result.

Acknowledgement

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