IDENTIFICATION OF SUB-SURFACE SOIL CHARACTERISTICS FROM SURFACE RECORDS

E. Şafak\textsuperscript{1} and E. Çaktı\textsuperscript{2}

ABSTRACT

The paper presents an approach to identify characteristics of layered soil media from surface records. It is assumed that the behavior of soil is linear and it is subjected to vertically propagating shear waves. The approach is based on the discrete time formulation of wave propagation, represented by upgoing and downgoing waves in the layers, and lattice filtering. For the identification of layer characteristics, we divide soil media into fictional thin layers, where the two-way travel time in each layer is equal to one sampling interval. With this model, we show that the relationship between the excitation and surface response can be represented by an auto-regressive moving-average type discrete time filter. Filter parameters are identified by using tools from system identification theory. Such filters can be converted into a cascade of lattice filters, each with two inputs and two outputs. Each lattice filter corresponds to a fictional soil layer defined by the reflection coefficient at the upper interface of the layer. Starting from the layer next to the surface, we can identify the reflection coefficients of the layers from the surface record. The reflection coefficients of those fictional layers that do not correspond to an actual interface would be identified as zero. If there are downhole records, we use them to crosscheck and confirm the identification results. Numerical examples are provided for the methodology.

\textsuperscript{1}Professor, Dept. of Earthquake Engineering, Kandilli Observatory and Earthquake Research Institute, Bogazici University, Istanbul, Turkey (email: erdal.safak@boun.edu.tr)

\textsuperscript{2}Professor, Dept. of Earthquake Engineering, Kandilli Observatory and Earthquake Research Institute, Bogazici University, Istanbul, Turkey (email: eser.caakti@boun.edu.tr)

Proceedings of the 11\textsuperscript{th} National Conference in Earthquake Engineering, Earthquake Engineering Research Institute, Los Angeles, CA. 2018.
Identification of Sub-Surface Soil Characteristics From Surface Records

E. Şafak\textsuperscript{1} and E. Çaktı\textsuperscript{2}

\textbf{ABSTRACT}

The paper presents an approach to identify characteristics of layered soil media from surface records. It is assumed that the behavior of soil is linear and it is subjected to vertically propagating shear waves. The approach is based on the discrete time formulation of wave propagation, represented by upgoing and downgoing waves in the layers, and lattice filtering. For the identification of layer characteristics, we divide soil media into fictional thin layers, where the two-way travel time in each layer is equal to one sampling interval. With this model, we show that the relationship between the excitation and surface response can be represented by an autoregressive moving-average type discrete time filter. Filter parameters are identified by using tools from system identification theory. Such filters can be converted into a cascade of lattice filters, each with two inputs and two outputs. Each lattice filter corresponds to a fictional soil layer defined by the reflection coefficient at the upper interface of the layer. Starting from the layer next to the surface, we can identify the reflection coefficients of the layers from the surface record. The reflection coefficients of those fictional layers that do not correspond to an actual interface would be identified as zero. If there are downhole records, we use them to crosscheck and confirm the identification results. A numerical example is provided for the methodology.

\textbf{Introduction}

The amplification (i.e., site amplification) of seismic waves by near-surface soil layers is a critical factor influencing the level of shaking experienced on ground surface (Safak, 1997). One of the most accurate ways to identify site amplification is to record ground motions synchronously at different depths below the surface. Such arrays are known as vertical downhole arrays. Downhole arrays are expensive to install because of drilling costs and the cost of downhole sensors.

For soil sites that can be approximated as layered soil media and subjected to vertically propagating shear waves, it is possible to identify and estimate the characteristics of sub-surface layers from the surface records alone (Ewing, et. al., 1957). This paper presents an approach for this based on discrete-time formulation of wave propagation in layered media. We show that the surface record can be represented as the response of an IIR (Infinite Impulse Response) filter, modeled in lattice form, and subjected to a Gaussian white-noise input.

\textsuperscript{1}Professor, Dept. of Earthquake Engineering, Kandilli Observatory and Earthquake Research Institute, Bogazici University, Istanbul, Turkey (email: erdal.safak@boun.edu.tr)
\textsuperscript{2}Professor, Dept. of Earthquake Engineering, Kandilli Observatory and Earthquake Research Institute, Bogazici University, Istanbul, Turkey (email: eser.cakti@boun.edu.tr)

Proceedings of the 11\textsuperscript{th} National Conference in Earthquake Engineering, Earthquake Engineering Research Institute, Los Angeles, CA. 2018.
**Discrete-time wave propagation in layered media**

Consider an $m$-layered media with input $X$ and the output $Y$, as shown in Figure 1a. It will be assumed that the layer $m+1$ has infinite depth such that the reflected waves from interface $m+1$ do not come back. The propagation of waves is defined by the wave travel times, $\tau$, in the layers, and the wave reflection and transmission coefficients, $r$ and $t$, for the upgoing waves, and $r'$ and $t'$ for the downgoing waves at each interface. The equations of wave propagation can be represented in terms of upgoing and downgoing waves, $U$ and $D$ at the top, and $U'$ and $D'$ at the bottom of each layer as shown in Fig. 1b. For the $i$'th layer, we can write the following (Claerbout, 1985):

$$D_{i+1}(t) = r' U_{i+1}(t) + t \cdot D'_i(t)$$
$$U'_i(t) = t' U_{i+1}(t) + r \cdot D'_i(t)$$

Note that

$$t = 1 + r, \quad r' = -r, \quad t' = 1 + r' = 1 - r,$$

$$U'_i(t) = U_i(t - \tau) = q^{-\tau_i} \cdot U_i(t) \quad \text{and} \quad D'_i(t) = D_i(t + \tau) = q^{+\tau_i} \cdot D_i(t)$$

Using these in the equations for layer $i$, we can write the following matrix equation for the relationship between the upgoing and downgoing waves at the top of the layer $i+1$ and layer $i$:

$$\begin{bmatrix}
U_{i+1}(t) \\
D_{i+1}(t)
\end{bmatrix} = \frac{1}{1 - r_i} \begin{bmatrix}
q^{-\tau_i} & -r_i \cdot q^{+\tau_i} \\
-r_i \cdot q^{-\tau_i} & q^{+\tau_i}
\end{bmatrix} \begin{bmatrix}
U_i(t) \\
D_i(t)
\end{bmatrix} = T_i \cdot \begin{bmatrix}
U_i(t) \\
D_i(t)
\end{bmatrix}$$

where

$$T_i = \frac{1}{1 - r_i} \begin{bmatrix}
q^{-\tau_i} & -r_i \cdot q^{+\tau_i} \\
-r_i \cdot q^{-\tau_i} & q^{+\tau_i}
\end{bmatrix} = \frac{1}{1 - r_i} \begin{bmatrix}
1 & -r_i \cdot q^{+\tau_i} \\
-r_i & q^{+\tau_i}
\end{bmatrix} = \frac{1}{1 - r_i} \begin{bmatrix}
q^{-\tau_i} & -r_i \cdot q^{+\tau_i} \\
r_i & q^{+\tau_i}
\end{bmatrix}$$
$T_i$ is known as the transfer matrix for layer $i$, for it transfers the upgoing and downgoing waves from the top of layer $i$ to the upgoing and downgoing waves to the top of layer $i+1$.

Since we do not know the layer interfaces, we will divide the media into superficial layers whose two-way travel time is equal to the sampling interval of the records; that is $2\tau_i = \Delta$, where $\Delta$ is the sampling interval. With this, the transfer matrix for layer $i$ now becomes:

$$T_i = \frac{q^{-\Delta/2}}{1 - r_i} \begin{bmatrix} 1 & -r_i \cdot q^\Delta \\ -r_i & q^\Delta \end{bmatrix} = \frac{q^{-\Delta/2}}{1 - r_i} \begin{bmatrix} 1 & -r_i / q^{-\Delta} \\ -r_i & 1 / q^{-\Delta} \end{bmatrix}$$

Figure 1. Layered medium: (a) Notation with input-bedrock and output-surface waves, (b) upgoing and downgoing waves at the top and bottom of layer $i$. 
Since the reflection coefficient for the surface (soil-air interface) is -1 for the downgoing waves, and +1 for the upgoing waves, we can write the upgoing and downgoing waves at the top of the first layer $U_1 = Y/2$, $D_1 = Y/2$. Also, noting that at the top of the bedrock we only have an upgoing wave, $X$, the relationship between the upgoing and downgoing waves at the top of the bedrock and those at the top of first layer can be expressed as the product of the transfer matrices from surface to bedrock (i.e., from layer 1 to layer $m+1$) by the following equation:

$$\begin{bmatrix} X(t) \\ 0 \end{bmatrix} = T_1 \cdot T_1 \cdots T_m \cdot \begin{bmatrix} Y(t)/2 \\ Y(t)/2 \end{bmatrix}$$

**Identification of sub-surface layers**

With the assumption that the media is composed of layers with two-way travel time is equal to one sampling interval, and inserting the expressions for layer transfer matrices, we can show that the relationship between $X(t)$ and $Y(t)$ is equivalent to an IIR (Infinite Impulse Response) filter of the following form:

$$Y(t) = \frac{b_0}{1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}} \cdot X(t)$$

where, for convenience, we have assumed that the sampling interval is $\Delta=1$ (i.e., $q^{-1}=q^\Delta$). It is reasonable to approximate the bedrock input waves, $X(t)$, as a Gaussian white-noise random process. With this assumption, the equation above represents an auto-regressive polynomial (AR) model for $Y(t)$. The coefficients $a_i$ in the denominator can be calculated by various methods available in the literature (e.g., Burg, 1968). It is possible to convert these coefficients into reflection coefficients and represent the AR model in the form of a cascade of lattice filters, as shown in Fig. 2 (Goodwin and Sin, 2009).

Each lattice filter represents the transfer matrix of a layer. By using the lattice filters, we can calculate the upgoing and downgoing waves for each successive layer, starting from layer 1, as shown below.

$$\begin{bmatrix} U_2(t) \\ D_2(t) \end{bmatrix} = \frac{q^{-0.5}}{1-r_1} \cdot \begin{bmatrix} 1 & -r_1 / q^{-1} \\ -r_1 & 1 / q^{-1} \end{bmatrix} \cdot \begin{bmatrix} Y / 2 \\ Y / 2 \end{bmatrix}$$

$$\begin{bmatrix} U_3(t) \\ D_3(t) \end{bmatrix} = \frac{q^{-0.5}}{1-r_2} \cdot \begin{bmatrix} 1 & -r_2 / q^{-1} \\ -r_2 & 1 / q^{-1} \end{bmatrix} \cdot \begin{bmatrix} U_2(t) \\ D_2(t) \end{bmatrix}$$

... ... ...

The recorded motion $Y_i(t)$ at interface $i$ is the sum of the upgoing and down going waves at the top of the layer $i+1$, that is: $Y_i(t) = U_{i+1}(t) + D_{i+1}(t)$. 

$Y(t) = b_0 / (1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}) \cdot X(t)$
Therefore, as we go down calculating the upgoing and downgoing waves at each layer, we also calculate the total response $Y_i(t)$ at each interface, and continue until the upgoing wave at the top the layer below is approximately a white noise and the downgoing wave is zero.

Figure 2. Lattice filter representation of upgoing and downgoing waves in layered media.
Example

We present an example for the methodology presented. Consider the three-layer soil media with properties given in Fig. 3. The input bedrock accelerations and the calculated surface accelerations are shown in Fig. 4. We match an AR model to the surface accelerations and calculate the reflection coefficients of the corresponding lattice filter. The reflection coefficients are plotted in Fig. 5 against the half-sampling points starting from the surface. The half-sampling points that correspond to large amplitude reflection coefficients (points 80, 100, 140) clearly indicate the layer interfaces, and match the locations given in the table in Fig. 3.

Figure 3. Properties of 3-layer soil media used in the example.
Figure 4. Bedrock and surface accelerations for the soil media in Fig. 3.

Figure 5. Reflection coefficients that match the surface record of the 3-layer media.
Conclusions

For layered soil media subjected to vertically propagating shear waves, it is possible to estimate the characteristics of sub-surface layers (i.e., layer interfaces, wave travel times, and reflection coefficients at the interfaces) from the surface records. For this, we divide soil media into fictional thin layers, where the two-way travel time in each layer is equal to one sampling interval. With this model, we show that the relationship between the excitation and surface response can be represented by an auto-regressive moving-average type discrete time filter. Filter parameters are identified by using tools from system identification theory. Such filters can be converted into a cascade of lattice filters, each with two inputs and two outputs. Each lattice filter corresponds to a fictional soil layer defined by the reflection coefficient at the upper interface of the layer. An example shows the application of the methodology.

References


