TOPOLOGY OPTIMIZATION OF ELASTIC SPINES IN ROCKING BRACED FRAMES

A. Martin¹ and G. G. Deierlein²

ABSTRACT

Rocking spine systems with energy-dissipating devices are innovative earthquake resistant structural systems that can maintain near zero-residual drift though self-centering action. In such systems, the rocking spines consist of steel braced frames or other structural systems (e.g., concrete walls) that are designed to remain elastic, while all the inelasticity is concentrated in energy-dissipating elements of an articulated rocking hinge. Building on previous research to develop practical ways to design rocking frame systems through surrogate models, this research explores the use of topology optimization for capacity design of elastic rocking spines. Techniques such as the modified modal superposition (MMS) can be calibrated to provide realistic estimates of the force demands in the elastic spine, which in turn can be utilized for structural topology optimization of the spine. The classical topology optimization approach to minimize compliance is extended for multiple load cases for the surrogate nonlinear mode(s) and elastic modes. Numerical results demonstrate important conceptual design considerations for rocking braced frames.

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Introduction

Self-centering rocking systems provide enhanced performance in the event of strong earthquakes by concentrating inelasticity in energy dissipation components and eliminating residual drifts using post-tensioning cables as self-centering elements [1, 2]. These systems are comprised of three main components: (1) an elastic spine consisting either of steel braced frames (chevron or X-bracing) or reinforced concrete shear walls, (2) an energy dissipation component such as steel shear fuses, buckling restrained braces, or viscous dampers and (3) post-tensioning cables to maintain an upright position after the earthquake (Figure 1). The rocking mechanism helps limit the induced earthquake forces in the elastic spine, and the energy dissipation components damp out the rocking displacements. Since the inelasticity and damage is concentrated at the base rocking joint, the steel frame members are designed to remain elastic. While the rocking base limits the first-mode response, it does not limit the higher modes, which can result in large force demands in the members [3]. This research aims to create more efficient rocking frames by applying structural topology optimization to determine the optimal geometric configuration of the elastic spine of

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rocking braced frames. Surrogate models are used to approximate the nonlinear dynamic response in the topology optimization process.

Figure 1. Self-centering rocking systems (a) rocking braced frame and (b) rocking shear wall.

**Topology Optimization Framework**

Topology optimization is a mathematical programming technique to determine the optimal distribution of material for a given boundary value problem. Widely used in aerospace and mechanical engineering, this technology has attracted interest in civil engineering, including design applications high-rise buildings under wind load [4]. Most research and applications of topology optimization to date consider only linear elastic structural behavior. Dynamic and nonlinear effects would need to be incorporated for earthquake engineering applications.

Previous research on the capacity design of rocking braced frames has shown that the modified modal superposition (MMS) [5], among other modified modal analyses, yield accurate prediction of forces in these systems by accounting for the reduction in the first mode response [6]. These surrogate methods take advantage the concentration of nonlinearities in one or more discrete rocking locations. In rocking frames with a single rocking hinge, most of the nonlinear rocking response occurs in the first vibration mode. Such systems can be idealized as a combination of an equivalent nonlinear first mode with elastic higher modes. Based on this observation, the MMS is used as a surrogate elastic analysis in a standard nested formulation for topology optimization, based on minimum compliance (maximum stiffness). The approach treats the multi-mode response as multiple load cases for topology optimization, described as follows:

minimize \[ C(\rho) = \sum_{i=1}^{m} \alpha_i F_i^T u_i \]  
subject to \[ \int_{\Omega} \rho dV - \bar{V} \leq 0 \]  
with \[ 0 \leq \rho_e \leq 1, \quad \forall e \in \Omega \]  
\[ F_i = Ku_i, \quad \text{for } i = 1, ..., m \]

where \( C \), the objective function, is a weighted-average compliance based on the modal forces, \( \rho \), is the design variable of element densities, \( F_i, u_i \) and \( \alpha_i \) are the modal forces, displacements and weights, respectively. \( \bar{V} \) represents the maximum volume and \( K \) is the global stiffness matrix.
Loading and Boundary Conditions

The modal forces applied to the rocking frame are analytically derived from a cantilever shear beam analogy [7], where the first rocking mode is an inverted triangular distribution, given as:

\[ F_1^n = \frac{S_a(T_1)}{R} \frac{W_{trib}}{N} \left( \frac{H_n}{H} \right) \]  

(2)

\[ F_2^n = 0.569 S_a(T_2) \frac{W_{trib}}{N} \sin \left[ \frac{4.49 H_{mid}^n}{H} \right] \]  

(3)

\[ F_3^n = 0.229 S_a(T_3) \frac{W_{trib}}{N} \sin \left[ \frac{7.73 H_{mid}^n}{H} \right] \]  

(4)

where \( F_i^n \) represents the modal force at story \( n \) from mode \( i \), \( S_a \) represents the spectral acceleration at the period \( T_i \), \( W_{trib} \), \( H \), and \( N \) represent the tributary weight, height and number of stories, respectively, and \( H_n \) and \( H_{mid}^n \) represent height and the mid-height of floor \( n \). A strength reduction factor, \( R \), is introduced to account for the nonlinearity in the first mode. The first three modes are deemed sufficient to account for the dynamic loading. The post-tensioning is idealized as a linear spring at the top of the structure. In addition, to simulate the rocking motion, the boundary conditions of the first mode are modified by removing one of the pinned base supports and adding post-tensioning (PT) and energy-dissipating (ED) loads. Both right and left rocking modes are considered to enforce symmetry (Figure 2). The modal weights \( \alpha_i \) are 0.5, 0.5, 1 and 1 respectively.

Figure 2. Rocking braced frame modal boundary conditions (a) right rocking mode 1, (b) left rocking mode 1, (c) mode 2 and (d) mode 3.

Optimal Rocking Braced Frames

A building prototype of 12 stories, \( H = 51.2 \) m by \( 2B = 9.14 \) m is considered for this study. Tributary masses are assumed to connect to the rigid spine every two floors on both sides. The steel structure has a modulus of elasticity of \( E = 200 \) GPa. The earthquake force demands are determined for a building site in San Francisco with a strength reduction factor of \( R = 8 \). A volume fraction of 45% and a projection radius of \( r_{min} = 2.5 \) were used for the optimization [8]. The optimization results are shown in Figure 3 under the induced earthquake forces compared to the traditional wind load case for a fixed-base high-rise building [4]. The braces are clearly defined in the final geometry. Due to base overturning forces developed by the from PT and ED, the column sections are large. The optimized geometry differs drastically from the traditional wind load case, where the \( \frac{3}{4} \) bracing point rule was determined as optimal. For rocking braced frames under earthquake loading, an elliptical bracing pattern is obtained to account for higher mode effects.
In this study, topology optimization framework was extended for multiple modal load cases for optimizing the bracing layout of rocking braced frames. Combining a surrogate nonlinear dynamic analysis such as the MMS method yields a continuous stress distribution in the members accounting for higher mode effects. This study shows the potential for earthquake engineering applications of topology optimization through modified modal analyses.

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References