FIRST MODE DAMPING RATIOS INFERRED FROM THE SEISMIC RESPONSE OF BUILDINGS

C. Cruz¹ and E. Miranda²

ABSTRACT

This paper analyzes damping ratios inferred from 1335 seismic responses, obtained from 154 different buildings in California. All the damping ratios were inferred using a parametric system identification technique in the time domain, and subjected to a series of reliability screening tests to ensure that only high-quality data was included. The resulting damping ratios conform a data set of 1037 high-quality values inferred exclusively from the seismic response of buildings, a database several times larger than all previous studies on damping. The data set is analyzed using a linear mixed effects statistical model to account for the fact that many of the data points come from damping ratios in the same building shaken by various earthquakes and then a conventional regression analysis, which assumes independence of all data points, may not be employed to perform statistical inference on the data. It is shown that damping decreases with increasing building height, which is the factor that best explains the relatively large variance observed in the data. Results show that, after correcting by building height, there is no statistical significance between the damping ratios of reinforced concrete and steel buildings. However, when including the combined material and lateral resistant system as a factor in the statistical model an additional 6% of the variance is explained. Results show that steel buildings with moment-resistant frames have, on average, a slightly higher damping ratio than those with steel braced frames. The amplitude dependency of damping is examined, showing that the overall lateral deformation demand in the building, as measured by the peak roof drift ratio, has little effect in damping ratios subjected to moderate earthquakes.

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Introduction

In engineering practice, the seismic design of a building is typically done employing an elastic response spectrum analysis using modal superposition. This requires knowing the building’s modal periods, mode shapes, modal participation factors, and modal damping ratios. The periods, mode shapes, and participation factors can be computed from the mechanical properties of the building, such as its distribution of mass and stiffness. However, a similar procedure cannot be used to estimate the level of damping because there are several sources of energy dissipation involved, and there is limited knowledge about them. Given the wide variety of different sources that contribute to energy dissipation, it is impractical to build models for damping based on component behavior. Moreover, it is not possible to infer realistic damping values in laboratory experiments – even when conducting dynamic tests on a shake table –

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because these tests do not reproduce the boundary conditions of real buildings. For example, radiation damping, which is often the largest source of energy dissipation in a building, cannot be reproduced realistically in a laboratory. Therefore, the most reliable way to estimate the level of damping in a building is through measurements of its actual structural response when subjected to dynamic loads. Given the impossibility to directly calculate the level of damping of a building, structural engineers must rely on empirical damping recommendations.

Since the late 1950s there have been several damping recommendations for the design of new buildings (a detailed literature review of their development over the years can be found in [1]). The very first recommendations were based on equivalent damping ratios obtained from static tests on a relatively small number of elements and connections tested in various laboratories and somehow extrapolating those results to structures employing the same materials ([2–4]). The next generation of damping recommendations came from statistical analyses of databases of damping values inferred from vibrations measured in structures when subjected to different sources of loading (e.g., [5–11]). In many cases, new recommendations were made after just adding a few data points to previous data sets. Practically all these recommendations share at least one of the following problems: (1) The methods employed to infer the damping ratios are now known not to be reliable; (2) the databases contain almost exclusively low-amplitude motions (e.g., ambient vibrations or low-amplitude forced vibrations) which in most cases are not valid for seismic analysis; (3) the databases contain more than one data point per building, but the regressions are not properly adjusted to take this into account.

The objective of this paper is to investigate damping ratios in the first mode of vibration of buildings and to provide new improved recommendations that avoid the aforementioned shortcomings. This paper provides recommendations for the damping ratio that a structural engineer should use for the first translational mode when performing a linear elastic modal analysis with a fixed-base model. To this end, a database of 1037 damping ratios was constructed after analyzing a total of 1335 seismic responses coming from 154 buildings and 117 earthquakes. The measured seismic responses of instrumented buildings were obtained from the Center for Engineering Strong Motion Data (CESMD), a cooperative center that integrates the earthquake strong-motion data collected by the US Geological Survey (USGS) and the California Geological Survey (CGS). All the damping ratios included in the database were carefully examined, after passing a series of reliability screening tests. Damping recommendations were then developed after a comprehensive statistical analysis of the data, where the influence of the building’s height, aspect ratio, first translational period, primary structural material, lateral-load resistant system, and the amplitude of the recorded seismic response was carefully examined.

**System Identification**

System identification is a tool that permits obtaining information about a system from its measured dynamic response. Specifically, a parametric system identification procedure consists in finding the set of model parameters that minimizes the difference between the response recorded by sensors installed in a building and the one predicted by a numerical model of the building. In this investigation, a modal minimization technique in the time domain was employed (e.g., [12]). This section provides a brief explanation of the mathematical model used, and the setup of the optimization problem.

The system identified in this study corresponds to a structure with its mass lumped at the different floors, with classical viscous damping, and a fixed-condition at its base i.e., with no
explicit modelling of the soil-structure interface. It is important to mention that, under these assumptions, the properties identified do not correspond to those of the superstructure alone—what is sometimes referred to as the “inherent structural damping”, but rather they represent the damping ratio of the combined soil-foundation-structure system. This is often referred as the equivalent fixed-base model of the building and its dynamic properties are referred as the “effective” properties of the model [13]. Therefore the inferred damping ratios correspond to those that would best reproduce the measured response of the building when using a fixed-base linear time-history response analysis. This was achieved by using a modal minimization method for system identification in the time domain. This identification method seeks for the modal periods, damping ratios, and mode shapes that minimize the difference between the relative acceleration $\ddot{u}(t)$ measured by the different sensors in the building, and that predicted by a fixed-base linear modal time history analysis $\ddot{\hat{u}}(t)$. This was done by minimizing the following objective function:

$$J(\theta) = \sum_{j=1}^{N_{sen}} \sum_{i=1}^{\tau} \frac{[\ddot{u}_j(i\Delta t) - \ddot{\hat{u}}_j(i\Delta t)]^2}{\sum_{k=1}^{\ell} [\ddot{u}_j(k\Delta t)]^2}$$

where $N_{sen}$ is the number of sensors above ground level; $\Delta t$ and $\tau$ are the time step and the number of points in the signal, respectively; and $\theta$ represents the parameters to identify/infer, which if the structure is assumed to be at rest at $t = 0$ are given by:

$$\theta = \left( \begin{array}{c} \{T_1 \} \\ \vdots \\ \{T_N \} \\ \{\xi_1 \} \\ \vdots \\ \{\xi_N \} \\ \Gamma_1\phi_{11} & \cdots & \Gamma_N\phi_{N1} \\ \vdots & \ddots & \vdots \\ \Gamma_1\phi_{1N_{sen}} & \cdots & \Gamma_N\phi_{NN_{sen}} \end{array} \right)$$

where $T_n$ and $\xi_n$ are the modal period and damping ratio of the $n$-th mode, and $\Gamma_n\phi_{nj}$ is the product of the modal participation factor and the mode shape of the $n$-th mode evaluated at the level where the $j$-th sensor is located.

Once the modal parameters of the building were identified, the reliability of the damping estimates was assessed by performing a series of reliability screening tests. The details of these tests, as well as additional information on the optimization problem can be found in reference [14].

Buildings Analyzed

A total of 1335 seismic recorded responses, measured in 154 different buildings located in California were analyzed. All the seismic records were obtained from the Center for Engineering Strong Motion Data [15]. After applying the reliability screening tests to the identification results, a database of 1037 damping ratios deemed to be reliable, obtained from 144 buildings, was assembled. Each entry of the database contains the identified damping ratio and period of the first translational mode, the earthquake that originated the data, the referential direction of analysis (NS or EW), the building’s height, aspect ratio, primary structural material, and lateral load-resistant system. Table 1 shows the number of buildings per primary structural material type in the data set, while Figure 1 shows a histogram of the building heights, grouped by the building primary structural material. It can be seen that most buildings (78%) are less than 50 m tall, but
there is still an important number of buildings above this height – 31 out of 144 (22%) – of which 16 are made of structural steel (51%), 8 of reinforced concrete (26%), and 7 (23%) have mixed primarily structure materials. The lateral load-resistant system of the buildings was classified as shear walls (SW), moment frames (MF), or braced frames (BF). The interested reader is referred to [1] for the full details of the building dataset.

<table>
<thead>
<tr>
<th>Material</th>
<th>Number of buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>54</td>
</tr>
<tr>
<td>Steel</td>
<td>58</td>
</tr>
<tr>
<td>Mixed</td>
<td>17</td>
</tr>
<tr>
<td>Masonry</td>
<td>9</td>
</tr>
<tr>
<td>Wood</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 1  Histogram of building heights, grouped by material

Regression Analysis

Linear Mixed Effects Models

If multiple damping ratios are measured from the same building, in the same direction, during different earthquakes, then these observations cannot be considered to be independent from each other. Consequently, any statistical inference made from a simple linear regression model would not be valid or could be biased/influenced by some buildings having produced more data points than others. A linear mixed-effects (LME) model solves this issue by separating the variability into two components, the variability between buildings (namely, the fixed effects of the statistical model), and the variability within buildings (namely, the random effects of the statistical model). In this investigation, a “building component” is defined as one of the two principal directions of a building. Results coming from two different perpendicular directions of the same structure were considered independent from each other, therefore each building generates the data of two different building components. The data was grouped by their respective building components, and the “building component” classification factor was treated as a random effect, which allows to take into account the variability within the damping ratios coming from the same building component. All the other factors (building height, aspect ratio, period, primary structural material, etc.) were regarded as fixed effects.
The specific LME model used in this investigation was the following:

\[
\ln(\xi_{ij}) = \beta_0 + b_i + \sum_{k=1}^{N} \beta_k X_{ki} + \epsilon_{ij}
\]

\[
b_i \sim N(0, \sigma_b^2) \quad \epsilon_{ij} \sim N(0, \sigma_\epsilon^2 \delta_k^2)
\]

where \(\xi_{ij}\) denotes the \(j\)-th observation of the damping ratio of the \(i\)-th building; \(\beta_0, \beta_1, ..., \beta_N\) are the coefficients of the fixed effects, \(X_{ki}\) are the known fixed-effects regressors, \(b_i\) are the random effects, and \(\epsilon_{ij}\) are the within building errors or residuals. The random effects are assumed to be normally distributed with zero mean and variance \(\sigma_b^2\). The residuals are assumed to be normally distributed with zero mean and variance \(\sigma_\epsilon^2 \delta_k^2\). This allows to consider different variances between levels of categorical variables (like material or lateral resistant system), where \(\delta_k\) represents the ratio between the standard deviations of the \(k\)-th stratum and the first stratum of the categorical variable. Note that \(k = \{2, ..., S\}\), where \(S\) is the total number of strataums. The logarithmic transformation for \(\xi\) was done to satisfy the linear and the within-category homoscedasticity assumptions of the model. This is equivalent to assume that the damping ratio is lognormally distributed. If only 1 categorical variable is present in the model, an overall measure of the dispersion in the model for the \(k\)-th stratum can be obtained as:

\[
\sigma_{ln_k} = \sqrt{\sigma_b^2 + \sigma_\epsilon^2 \delta_k^2}
\]

where the \(ln\) subscript is added to emphasize that this is the logarithmic standard deviation of damping ratio, coming from the logarithmic transformation of the data. Additional details about linear mixed-effects models can be found in reference [16], information regarding the computational implementation of these models can be found in reference [17].

**Model Selection**

The choice of the different factors to include as regressors for the fixed effects of the statistical model was done by measuring the percentage of the variance in the damping ratios explained by each regressor. The percentage of variance explained was quantified using the marginal coefficient of determination \(R^2\) for LME models developed by Nakagawa and Schielzeth [18], assuming constant variance for the residuals. The following subsections describe the different statistical models employed to analyze the data. The mathematical description of these models and their corresponding \(R^2\) values are listed in Table 2

**Parameters Related with Soil-Structure Interaction**

Previous studies have shown that there is a strong correlation between damping ratio and the building height \(H\), the first translational period \(T\), and the aspect ratio \(AR\) of the building (e.g., [9–11,19]). It has been argued that this correlation occurs because these parameters determine the degree of soil-structure interaction of the building (e.g., [11,20]). To investigate the influence of these factors, three different LME models considering only \(H\), \(T\), or \(AR\) as fixed effects were fitted to the data. It was found that the parameter that best explained the variance of the data was the building height, with an \(R^2\) of 0.43. These three parameters are highly correlated, so the fit does not significantly improve when combinations involving two or more of these regressors are included. Therefore, only the building height was considered in the
subsequent statistical models.

Table 2 Model selection: regression models with different fixed effects. All models consider the “building component” classification factor as a random effect.

<table>
<thead>
<tr>
<th>Regression Model</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Parameters related with soil-structure interaction:</td>
<td></td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) ]</td>
<td>0.43</td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(T) ]</td>
<td>0.20</td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(AR) ]</td>
<td>0.31</td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(T) ]</td>
<td>0.46</td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(AR) ]</td>
<td>0.43</td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(T) + \beta_0 \ln(AR) ]</td>
<td>0.46</td>
</tr>
<tr>
<td>2. Building height, and amplitude dependency:</td>
<td></td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(\text{PRDR}) ]</td>
<td>0.47</td>
</tr>
<tr>
<td>3. Building height, and material:</td>
<td></td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \sum_{k=2}^{M+1} \left( \beta_k D^\text{mat}<em>{(k-1)} + \beta_k + M D^\text{mat}</em>{(k-1)} \ln(H) \right) ]</td>
<td>0.45</td>
</tr>
<tr>
<td>4. Building height, and coupled material and lateral resistant system:</td>
<td></td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \sum_{k=2}^{Q+1} \left( \beta_k D^\text{MLS}<em>{(k-1)} + \beta_k + Q D^\text{MLS}</em>{(k-1)} \ln(H) \right) ]</td>
<td>0.49</td>
</tr>
<tr>
<td>5. Building height, amplitude, and coupled material and lateral resistant system:</td>
<td></td>
</tr>
<tr>
<td>[ \ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(\text{PRDR}) + \sum_{k=3}^{Q+2} \left( \beta_k D^\text{MLS}<em>{(k-2)} + \beta_k + Q D^\text{MLS}</em>{(k-2)} \ln(H) \right) ]</td>
<td>0.53</td>
</tr>
</tbody>
</table>

**Amplitude Dependency**

The measure of amplitude used in this study was the peak roof drift ratio (PRDR), defined as the peak displacement at roof level relative to the ground, normalized by the building height. When correcting by PRDR, as described in model 2 of Table 2, only an additional 4% of the variance was explained, reaching an \( R^2 \) value of 0.47. This small increase in \( R^2 \) suggests that the variation in damping ratio with changes in the overall lateral deformation of the building is small. Please note that the analysis of a building with an amplitude-dependent damping ratio would require iterations because PRDR and \( \xi \) would be mutually dependent. It was considered that the increase in model complexity and additional computational cost from an iterative analysis was not justified by the marginal gain in the accuracy of the model, so PRDR was not considered as a variable in the damping recommendation formula.

**Primary Structural Material**

The effect of the primary structural material was then incorporated to the statistical model. Mathematically, this corresponds to model 3 shown in Table 2, where \( D^\text{mat}_{k} \) is a dummy variable equal to 1 for buildings made of the \( k \) -th level of the Material categorical variable, and equal to 0 otherwise; where \( k = 1, \ldots, M \), and \( M \) is the number of different structural materials considered, minus 1. It was found that adding the primary structural material only explained an additional 2% of the observed variance in the damping ratios with respect to the basic statistical model that contains only the building height, resulting in an \( R^2 = 0.45 \). The low influence of the building structural material suggests that, contrary to most previous recommendations, the structural material is not a major determinant on the level of damping in
Combined Primary Structural Material and Lateral Force Resisting System

Another LME model incorporating the combined effect of structural material and lateral-force-resisting system, included in the model as the MLS categorical regressor, was created. As before, the mathematical formulation of the model is shown in Table 2, where $D_k^{MLS}$ is a dummy variable equal to 1 for buildings made of the $k$-th level of the MLS categorical variable, and equal to 0 otherwise; where $k = 1, \ldots, Q$, and $Q$ is the number of combined material-lateral-system categories considered, minus 1. It was found that this model improved the fit, reaching an $R^2$ value of 0.49. Suggesting that MLS is a better explanation for the variance than the building primary structural material itself.

Finally, a statistical model considering the building height $H$, the combined material and lateral-load resisting system MLS, and the peak roof drift ratio of the structural response $PRDR$ was created for verification purposes. The objective of this model was not to provide a damping prediction formula, but to verify that the statistical significance of the coefficients of the previous model was still valid when correcting for the amplitude of the response. Note that in this model the percentage of variance explained reaches 53%.

Regression Results

The following subsections summarize the results obtained for the different models discussed above. Due to space limitations, only the most relevant results are discussed. The interested reader is referred to [1] for the full details of this investigation.

Effect of the Building Height

The building height $H$ is the factor that explains the largest percentage of the variance in the observed damping ratios. Therefore, the simplest damping model uses only $H$ as a regressor (model 1 in Table 2). For illustration, the data, the predicted median, and the $±1\sigma_{ln}$ confidence intervals, as well as the equation to compute the predicted median values, are shown in Figure 2. As expected, there is a strong negative correlation between the two variables, as it can be seen that damping decreases with increasing building height following a hyperbolic trend. For buildings lower than 21m the median damping ratios are higher than the typical 5% recommended in most current structural codes. The opposite occurs for buildings taller than 21m, where the median damping is lower than 5%. Given that the selected functional form tends to infinity as $H$ approaches zero, this prediction equation is recommended for buildings above 10m tall.

Effect of the Primary Structural Material

The effect of the building’s primary structural material (model 3 in Table 2) was analyzed next. The most relevant result from this model is that no statistical difference was found between the data coming from concrete buildings and that coming from steel buildings, probably because the large variability around the median values. This is consistent with recent results by Smith et al. [21] on buildings subjected to low-amplitude wind vibrations. Figure 3 shows the relationship between damping ratios and building height, but deaggregated by those coming from steel and concrete buildings. It can be seen that there is no clear difference between the data coming from these two types of materials. This finding, and the small percentage of variance explained by including the building primary structural material in the model, suggests that other factors not included in the model, such as soil-structure interaction, may provide a better explanation for the
observed variability in damping ratios. Consequently, the single-regressor model of Figure 2 is recommended for the design of buildings of all materials.

![Figure 2](image1)

**Figure 2** Damping ratio as a function of building height and $\pm 1\sigma_{ln}$ confidence intervals

![Figure 3](image2)

**Figure 3** Damping ratios from steel and concrete buildings as a function of the building height.

**Combined Primary Structural Material and Lateral Force Resisting System**

The influence of the lateral-resistant system was investigated next by adding a combined material- and-lateral-resistant-system (MLS) term to the regression (model 4 of Table 2). It was found that on average, steel moment-frame have larger damping ratios than steel braced-frame buildings and that this difference is statistically significant. This is illustrated in Figure 4, which shows the relationship between damping ratios coming from steel buildings and the building height, deaggregated by lateral force-resisting system. It can be seen that the regression line corresponding to steel moment-frame buildings is above the regression line calculated for steel braced-frame buildings for all building heights. These findings are consistent with observations made on buildings subjected to wind motions (e.g., [22–24]). It has been argued that this difference occurs because the relative contribution of the shear deformations to damping is larger...
than that of flexural deformations [23]. Soil-structure interaction also explains this: if the same height and mass distribution is assumed for both buildings, then the overturning moment of the first mode is larger in braced-frame buildings than in moment frame buildings [25]. Therefore, soil-structure interaction, which is largely dominated by the first mode, will have a larger contribution of rocking motion in braced-frame buildings than in moment-frame buildings, leading to lower effective damping ratios in braced-frame than in moment-frame buildings [20].

![Figure 4](image)

Figure 4  Damping ratios from steel moment-frame and steel braced-frame buildings as a function of the building height.

**Conclusions**

This work evaluated damping ratios for the fundamental mode obtained from the analysis of 1335 seismic responses of instrumented buildings in California. All the damping ratios were inferred using a parametric modal minimization in time, and subjected to a series of reliability screening tests to ensure that only high-quality data was employed in the statistical analysis. The dataset was analyzed using a linear mixed-effects (LME) statistical model was employed. It was shown that the factor that best explains the variance in the data is the building height. A series of more complex, multi-variate statistical models were also evaluated. Results show that, in terms of percentage of variance explained, there is little gain in favoring a more complex model over a simple height-dependent prediction equation. Therefore, a single prediction equation that relates damping in the fundamental mode with the building height was presented. It was observed that the damping ratio of the fundamental mode decreases with increasing building height following a hyperbolic trend. It was found that the amplitude of the response, measured in terms of the peak roof drift ratio, explained only an additional 4% of the variance, suggesting that the variation in damping ratio with changes in the overall lateral deformation of the building is small. The influence of the building material and lateral-load resistant system was then investigated. Results showed that there is no statistical significance between the damping ratios of reinforced concrete and steel buildings. Regarding lateral-load resistant systems, it was found that steel moment-resistant frames have larger damping ratios than steel braced-frame buildings. All these findings suggest that soil-structure interaction is the factor that primarily governs the overall damping ratio of a building.
References