DIRECTIVITY CENTERING OF GMPES AND OF DIRECTIVITY MODELS

B. Rowshandel

ABSTRACT

To account for near-source effects, directivity models are often used in combination with conventional, “non-directive” Ground Motion Prediction Equations (GMPEs) to calculate directivity amplification factors. The validity of this approach depends on “centering” of both the GMPE and the directivity model used. Recorded ground motions naturally have some contributions from rupture directivity, causing the GMPEs that use such data to carry some average directivity effects in them. Centering of GMPEs, from directivity point of view, deals with how large this inherent average directivity is, and to what extent it is unbiased. Directivity centering in GMPEs depends on many aspects of the selected data which goes into developing the models, specifically the azimuthal locations of the recording stations, the geometry and style of faulting, and the ground motion period. When using directivity models to either transform a non-directive GMPE to a directivity-capable one, or to find directivity amplifications, this implicit average directivity should be correctly accounted for by properly adjusting the target directivity model.

A methodology for investigating centering in non-directive GMPEs is first presented. Using the methodology, directivity-centering of the NGA-West2 data and centering of the five GMPEs at different ground motion periods, were investigated. The results indicate that, with exception of one GMPE, the NGA-West2 GMPEs can be considered centered for periods up to 5 seconds.

Next, the approach presented is used to center the directivity model developed by the author in the NGA-West2 project. The directivity model was centered using the average directivity observed in the NGA-West2 near-source data combined with simulation data obtained for a range of test cases. Simulation data were used in conjunction with empirical data to capture variabilities in fault geometry, style of faulting, and hypocenter locations that are not adequately represented in the NGA-West2 dataset; and hence extend the applicability of the model to GMPEs beyond those developed in the NGA-West2 project.

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Introduction

The efforts in developing Directivity Models in the Next Generation Attenuation (NGA-West) project involved fitting directivity functional forms, or directivity parameters to residuals of Ground Motion Prediction Equations (GMPEs). The goal was to produce directivity functional forms that could be used as post hoc corrections to the medians of the GMPEs for average horizontal or directional ground motions. Two such models were developed based mainly on the NGA-West1 data [1]. In the Next Generation Attenuation-West2 (NGA-West2) project, the original goal was for the directivity modelers to develop directivity functional forms which the

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GMPE developers could include as a component of the GMPEs before conducting their numerical regressions. Five directivity models were developed in the NGA-West2 project. However, as the project was approaching its completion, following an assessment of the models, and mainly based on the recommendations from the Directivity Working Group, it was generally determined that the directivity models were not sufficiently developed to be included in the GMPEs. As a result, with exception of Chiou and Youngs, who incorporated their directivity model into their GMPE (2014), developers decided not to include any directivity term into their models. After the completion of the NGA-West2 project a “Directivity Panel,” composed of expert users and with consultation and support from Directivity Modelers and GMPE developers, was assembled to continue to explore the subject, coordinate activities among directivity and GMPE modelers, and make short-term and long-term recommendations [2]

One major issue with the NGA-West2 directivity models was the large variability among their predictions for non-strike-slip faults. This variability among model predictions was mainly due to the assumptions made by different modelers, prompted in part by inadequate information on directivity in the empirical data. Any improvement in the models therefore required constraining the models using additional data. This model-to-model variability resulted in a recommendation from the Directivity Working Group to use more than one model for non-strike-slip faults to capture the epistemic uncertainty. One major issue, not adequately addressed at the completion of NGA-West2 project, was the extent to which the data and the models were unbiased (i.e., “centered”) with respect to directivity. Since there were no studies on the implementation of NGA-West2 directivity models into Seismic Hazard Analysis (SHA) codes at that time, the impact and implications of the directivity models on ground motion hazard also needed investigation.

One of the five NGA-W2 Directivity Models was developed by the author of this article [3, 4]. The issue of centering of the model was addressed to some extent during the development of the model, but time constraints did not allow a comprehensive treatment of the subject within the framework of the NGA-West2 project. In addition, the model had not been implemented into any SHA code, so its impacts and implications were not studied. There has been significant improvement of this model since the completion of the NGA-West2 project. Recent work, completed and in progress, consists of: (i) improving the “narrow band” features of the model [5], (ii) more detailed work on centering, summarized in this paper, (3) improving the capability of the model in treating heterogeneous rupture and slip, and (4) implementation in SHA code and study of its implications.

The Directivity Model

The directivity model described in this study was developed based on the premise that maximum positive ground motion directivity at a site occurs when the directions of rupture and slip on the fault are both toward the site. The directivity parameter $\zeta$ for the case of homogeneous rupture and slip, is presented in Eq. 1 and graphically illustrated in Fig. 1.

$$\zeta = (\zeta' - \zeta_c') \times DT \times W_p$$  \hspace{1cm} (1)

Where $\zeta'$ is the wide-band, rupture-length de-normalized directivity parameter before applying any tapers, $\zeta_c'$ is the directivity-centering parameter, $DT$ is the distance-taper, and $W_p$ is the narrow-band multiplier. Details of the model can be found in the earlier work by the author [3,4].
As shown in Fig. 1, the directivity parameter is computed based on the direction of rupture and the direction of slip. The coordinates of the rupturing element of the fault plane, shown as the gray square, i.e., sub-fault \( i (X_i, Y_i, Z_i) \), and the coordinates of the site \((X_s, Y_s, 0)\) are used to compute the site unit vector, \( q \) along the vector connecting the sub-fault \( i \) and site. The coordinates of the hypocenter \((X^*, Y^*, Z^*)\), shown by the “star”, and the coordinates of the rupturing sub-fault \( i (X_i, Y_i, Z_i) \) are used to compute the rupture unit vector, \( p \) (along the vector connecting hypocenter and sub-fault \( i \)). The unit slip vector, \( s \), for the sub-fault \( i \) is computed using rake angle \( \gamma \) and the information on the geometry of the fault. When the rake angle is assumed the same for the entire fault, \( s \) would not depend on the (location of the) sub-fault.

\[
s_x = \pm \left\{+ \sin(\gamma)\cos(\varphi)\cos(\theta) - \cos(\gamma)\sin(\theta)\right\}
\]
\[
s_y = \pm \left\{- \sin(\gamma)\cos(\varphi)\sin(\theta) - \cos(\gamma)\cos(\theta)\right\}
\]
\[
s_z = \pm \left\{- \sin(\gamma)\sin(\varphi)\right\}
\]

The choice of the +/- sign in front of the expressions above depends on the (along-the-strike and across-the-dip) location of the sub-fault relative to the location of the hypocenter. Using the unit vectors \( p, q, \) and \( s \), the rupture-based directivity parameter \( \xi_p' \) and the slip-based directivity parameter \( \xi_s' \) for the site are the sums of scaler products over the \( N \) sub-faults making up the fault:

\[
\xi_p' = (\Sigma p \cdot q) / N, \quad \xi_s' = (\Sigma s \cdot q) / N
\]

The directivity parameter for the site is taken as either \( \xi_p' \) or \( \xi_s' \) or a weighted combination of the two. The weighted combination, found to be most effective in capturing directivity, gives equal weights to the direction of rupture and the direction of slip ([3], Chapter 3):

\[
\xi' = 0.5 (\xi_p' + \xi_s')
\]
In the absence of information on the rake angle, \( \xi' \) can singly be used to model directivity effects, and when information on the rake angle is available and information on the location of the hypocenter is not, \( \xi'' \) can be used. An example of the former is when developing early real time shakemaps where preliminary information on hypocenter is available, but rake angle is not known. An example of the latter is when estimating ground motions for a future earthquake for which the location of rupture initiation (hypocenter) cannot be forecasted, whereas the direction of slip (rake angle) can be roughly inferred from information on dip angle and the regional stress.

The directivity parameter on the sub-fault is a purely geometrical parameter, fully specified by three sub-fault-dependent unit vectors \( p, q, \) and \( s \). The range of values for \( \xi, \xi'_p, \) and \( \xi''_s \) is \((-1 \text{ to } +1)\) or \((0 \text{ to } 1)\), depending on the choice of model [7]. In the former, sites located in the rupture and/or slip front have positive \( \xi'_p \), and/or \( \xi''_s \) and sites located behind the rupture/slip have negative values. \( \xi = +1 \) represents maximum forward directivity regardless of the choice of the model.

The centered directivity parameter for the site, \( \bar{\xi} \), once multiplied by a period-dependent directivity coefficient, \( C \), results in a directivity correction term. This correction term when added to a logarithmically expressed, centered, nondirective GMPE results in a directivity-corrected GMPE. The corresponding directivity (de-)amplification term is simply \( e^{C \bar{\xi}} \).

**GMPE Centering**

When developing a conventional no-directivity GMPE, even though no explicit account of directivity is made as part of the model development process, the directivity effect in the observed dataset finds its way into the median of the GMPE. Therefore, to make a correction for directivity in a non-directive GMPE, the reference directivity condition corresponding to that median motion needs to be known. If this reference condition is not addressed properly, directivity correction with respect to a wrong reference condition could shift the median of the GMPE and thus produce biased results. Potential directivity bias in NGA-West2 dataset was not systematically investigated by the GMPE developers or by the directivity model developers. To be included into the GMPEs, it was required that the directivity functions be properly centered, that is, the reference directivity condition for the directivity models be clearly defined. This task was not performed by most, if not all, directivity modelers.

What is meant in the present study, by a GMPE being directivity-centered, is simply that the dataset on which the GMPE is based be complete or balanced regarding its directivity information needs. Therefore, to have a directivity-unbiased GMPE, the data should be unbiased with respect to all hazard variables which affect directivity. For this to be the case, first, ground motion records sampled and used in developing a GMPE, specifically the recording stations, should (ideally) equally cover all azimuthal locations. But this is not enough! A set of azimuthally evenly distributed recording stations does not guarantee unbiased directivity in a GMPE developed based on such data. For example, a dataset with all earthquake hypocenters located at mid-length of faults would result in a directivity-biased GMPE even with uniform station coverage. Such a GMPE would, on average, under-predict (especially the long-period) ground motions since its database represents bilateral ruptures only. Conversely, ground motion records from stations in the forward directivity region would contain higher than average ground motions. Such data would be biased toward high directivity. One example of such a situation, described in [3], is the recordings of \( M_6 \), 1966 Parkfield earthquake. Ground motion records for this event in NGA-West2 dataset are from four stations all located in the forward directivity region, and thus naturally
recorded higher than average motions, especially at long spectral periods. A non-directive GMPE fit to the Parkfield data would fit them on the average, and hence significantly overestimate the median of $M6$ earthquake ground motions.

The fault features which determine the directivity at a site include the geometry of the fault, the orientation of the fault with respect to the site, and the directions of rupture and slip (e.g., hypocenter location and rake angle). For a database such as NGA-West2, with a wide range of fault geometries and styles of faulting (dip angle, rake angle, hypocenter locations), and multiple records for each earthquake, the factors leading to ground motion over prediction and those leading to under prediction would likely tend to cancel. This would be the case, for example, if the combination of hypocenter locations and the azimuthal distribution of recording stations would lead to centered station locations with respect to directivity.

To determine the degree to which the NGA-West2 database is centered with respect to directivity, we computed the distributions of the directivity parameter, $\xi'$ (Eq. 3a-c) within it range ($\pm 1$) for the NGA-West2 data (i.e., using reported hypocenters, rake angles, and station coordinates), and then compared it with a “simulated centered” distribution. Two sets of analyses were performed on the 77 earthquakes with finite fault information in the NGA-West2 database. The combination of near-source ($R<25km$) recording stations around the NGA-West2 faults with finite fault information and their reported hypocenter locations resulted in the directivity distribution (i.e., distribution of $\xi'$ within its range) for the NGA-West2 database. To come up with the centered distribution, we used 1km by 1km grids of near-source sites distributed all around each fault in the database and a uniform grid of hypocenters on the surface area of each fault.

Figure 2. Distributions of rupture-slip directivity parameter, $\xi'$, for a $M7.7$, 4-segment, $45^\circ$ dipping fault for three different slipping scenarios: normal ($\gamma=-90^\circ$), strike-slip ($\gamma=0^\circ$), and reverse ($\gamma=+90^\circ$). The 5km wide “racetrack” at 15 km ($12.5km< R_{cl} <17.5km$) is also shown in blue in the center image.

The analyses consisted of computing the three measures of directivity, $\xi'$, expressed in Eq. 3a-c, their means and their distributions within their ($\pm 1$) ranges, on various “racetracks” around the faults, at the recording stations for the NGA-West2 dataset, and at the grid of sites for the centered scenario. A racetrack is the collection of sites around the fault at nearly equal (source-to-site) distance. The concept is explained in detail in [3]. For illustration, Fig. 2 shows the plots of directivity parameter $\xi'$ (Eq. 3c) around a hypothetical fault due to a uniform distribution of hypocenters and three slip scenarios (strike-slip, reverse, and normal). Also shown in this figure
is the 15\text{km} racetrack (12.5\text{km} < R_c < 17.5\text{km}). The choice of racetracks would allow us to compute the azimuthally-averaged directivity parameters, and to study and capture distance dependency of the average directivity, if any, by comparing average values on racetracks at different distances. Comparing average directivity, real and simulated, on racetracks at different distances around all faults, revealed no significant dependency of average directivity on distance.

Racetrack computations resulted in distributions for the directivity parameters within their ranges (i.e., $\xi_p'$, $\xi_i'$, and $\xi'$ within $\pm 1$) for each of the events with finite fault information in the NGA-West2 database. As an example, results for the case of the fault associated with the 1989 Loma Prieta earthquake are shown in Fig. 3. Shown in this figure are the distributions of directivity parameters $\xi_p'$, $\xi_i'$, and $\xi'$ (Eq. 3a-c) within the range $\pm 1$ in the near-source region of this fault due to uniformly distributed hypocenters. Such a numerical simulation was repeated for all 77 events with finite fault information in the NGA-West2 database. Results for rupture-based directivity parameter ($\xi_p'$ in Eq. 3a) and rupture-slip-based directivity parameter ($\xi'$ in Eq. 3c) are presented in Fig. 4. Each curve in Fig. 4 corresponds to one NGA-West2 earthquake fault with a grid of hypocenters and a grid of sites within 25 km. The mean (solid black curve) and mean $\pm 1$ standard deviations (dashed black curves) are also shown. The solid black curves (for $\xi_p'$ on the left and for $\xi'$ on the right) represent the mean of the 77 distributions of directivity parameter for near-source region of NGA-West2 faults, corresponding to a “hypothetical centered” dataset. To determine to what extent the (real, reported) NGA-West2 data, and various subsets of data used by each GMPE developer are centered, we find the near-source distribution of “racetrack-averaged” $\xi'$ for each dataset and data subset and compare them to the black curves in Fig. 4. The racetrack-averaged directivity distributions for the NGA-West2 data (consisting of fault geometries, reported hypocenters, and station coordinates within 25 km of the faults), based on Eq. (3a) and (3c), once combined and normalized, are shown by solid red curves in Fig. 4.

The information presented in Fig. 4 indicates that to the extent that the red curves, which represent the overall near-source directivity features of NGA-West2 data (consisting of geometry, style of faulting, and locations of hypocenters and recording stations) are close to the solid back curves, the data is centered or is unbiased from directivity point of view. Deviations of the solid red curves from the solid black curves represent directivity bias in the reported data. Where the red curve is below the black curve the directivity parameter is under-represented in the empirical data and where the red curve is above the black curve, the data is over-representing the directivity.
As stated earlier, the near-source directivity distribution (solid red) curves in Fig. 4 are based on all data within 25 km ($R_{cl}$) of the faults in the NGA-West2 database. Each NGA-West2 GMPE developer team used a different data sampling scheme and came up with a different data subset. Also, the processing of the data, specifically the filtering, resulted in reduction in the number of records at long ground motion periods. Therefore, the directivity centering bias is expected to vary among the data subsets used in different GMPEs and to also change with ground motion period.

To examine the centering of various NGA-West2 data subsets, we computed the near-source directivity distributions (e.g., red curves in Fig. 4) corresponding to subsets of data at periods from 1 to 10 seconds for the five NGA-West2 GMPEs. Sample results at 1 sec. and 10 sec. periods, and based on rupture-slip directivity parameter ($\xi'$ in Eq. 3c), are shown in Fig. 5. Results for the five GMPEs (abbreviated as A&S, B&A, C&B, C&Y, and Idrs) are shown in different colors. Comparing the colored curves with the dotted black curve (the simulated, centered case), zones of directivity bias in different data subsets can be identified. As can be observed from the graphs, except for Idriss (Idrs), directivity biases in the remaining four GMPEs are roughly similar. This would be expected as the Idrs-GMPE is based on a much smaller subset of data compared to the other four. Another observation from comparison of directivity distributions at 1-10 second periods with the centered one in (the upper-right and lower-left images in) Fig. 5 is that the directivity biases only gradually change for periods from 1 to roughly 5 seconds, but beyond that, the deviations become drastic.

Figure 4. Distributions of the directivity parameter in the near-source regions of 77 NGA-West2 faults for hypothetical (centered) data and for real-reported data. Black solid and dotted curves are the mean and mean $\pm$ one standard deviation curves for the 77 sets of centered-simulations and the two solid red curves are the distributions corresponding to the reported data.

Directivity Model Centering

The analysis described in the previous section was concerned with centering of the datasets which form the basis of GMPEs, and particularly the five NGA-West2 models. Regardless of the degree of centering of the data (e.g. the closeness of red curves to black curves in Fig. 4), there will still be an average implicit directivity in the non-directive GMPEs. For a directivity correction term to be properly applied to any non-directive GMPE, this average directivity must be accounted for. This is done by ensuring that the directivity model first removes implicit directivity effects, captured by the target GMPE due to the choice of data. Directivity data bias or deficiencies in the
GMPE datasets that stem from inadequacies in the number and the locations of recording stations, manifested as the discrepancy between centered (black solid curves) and biased GMPE data (red solid curves), can be remedied by use of new data, empirical or simulated. The methodology presented in this study can help identify the gaps in directivity data in existing GMPEs and in improved data sampling for constructing unbiased datasets in the future.

The average directivity at a site, represented by the directivity centering term $\xi^c$ in Eq. 1, is the mean of the directivity parameter over the complete range of directivity-inducing parameters in the near-source region. However, in computing this centering term, the effects of various parameters, mainly the site-to-source distance, earthquake magnitude or fault size, and faulting mechanism need to be studied and accounted for.

![Figure 5. Directivity centering of different NGA-West2 GMPE data subsets (stations within 25km) for periods of 1 to 10 seconds based on the directivity parameter of equation (3c).](image)

To determine the centering term, $\xi^c$, the average of $\xi$ along racetracks of different distances from the fault (e.g., $R_{av}$=15km, 12.5km<$R<$17.5km) were computed. NGA-West2 earthquake records with finite fault information were used to compute the centering term. However, to extend the applicability of the model, and to more thoroughly study the impacts on the centering term, of various hazard variables, such as earthquake magnitude, fault geometry, and style of faulting, a number of hypothetical test cases were also used. The test cases consisted of the ones used in NGA-West2 Directivity Working Group [3] and some new ones, about 30 test cases in all. The test cases varied in geometry, style of faulting, and other seismological features. They included multi-segment faults, with length, width, dip angle, and rake angle changing from segment to segment. A uniform hypocenter distribution was used and many feasible rupture and slip scenario
were modeled.

In part of the study, NGA-West2 earthquakes with finite fault information were divided into three groups based on their faulting mechanism: (1) strike-slip ($\gamma \approx 0^\circ, \pm 180^\circ$), (2) dip-slip ($\gamma \approx +90^\circ$), and (3) normal-slip ($\gamma \approx -90^\circ$). The dip angles ($\phi$) associated with the three styles of faulting were as follows: $\phi = 75^\circ$-$90^\circ$ for strike-slip, $\phi = 30^\circ$-$45^\circ$ for reverse-slip, and $\phi = 45^\circ$-$60^\circ$ for normal-slip faults. Directivity parameters at grids of near-source sites surrounding the faults for uniform distribution of hypocenters were computed. Using the results, expressed in terms of directivity parameter ($\zeta'$) as a function of distance from the fault, the average directivity on racetracks were computed. Partial results for one of the test cases were presented earlier, in Fig. 2.

![Figure 6](image.png)

**Figure 6.** Racetrack-averaged directivity parameter ($\zeta'$, Eq. 3c) as functions of fault maximum rupture length for three styles of faulting: Strike slip ($As$: red curve and circles), reverse ($Ar$: green curve and diamonds) and normal ($An$: blue curve and triangles).

The racetrack-averaged directivity parameters ($\zeta'_{ave}$) presented in Fig. 6 represent the centering terms for faults of maximum rupture length $XL$, for three styles of faulting: strike-slip ($\gamma = 0^\circ$), reverse ($\gamma = +90^\circ$), and normal ($\gamma = -90^\circ$). In the above Figure, with $L$ representing total fault length and $W$ being the average fault width over all segments, both in km: $XL = \sqrt{L^2 + W^2}$. As can be seen in Fig. 6, the centering term $\zeta'_{ave}$ depends on the style of faulting. To study the nature of this dependence, racetrack-averaged directivity for $\gamma = \pm 30^\circ$, $\gamma = \pm 45^\circ$, and $\gamma = \pm 60^\circ$ were computed. Variations of racetrack-averaged directivity with rake angle for one of the test cases is shown in Fig. 7. Similar patterns in change of average directivity with rake angle were observed for all test cases. A combination of two cosine half-cycles, each of different amplitudes (with the smaller amplitude corresponding to reverse-slip and the larger one representing normal-slip) approximately models this dependence. By matching the racetrack-averaged directivities at $\gamma = 0^\circ$ (and $\pm 180^\circ$) and $\gamma = \pm 90^\circ$ from Fig. 7 with the results presented in Fig. 6, the following expressions for the directivity centering term $\zeta_c'$, to be used in Eq. 1, were obtained:

$$
\zeta'_c = \frac{(As+Ar)}{2} + \frac{(As-Ar)}{2} \cos(2\gamma), \quad \text{reverse slip: } 0^\circ < \gamma < 180^\circ \quad (4a)
$$

$$
\zeta'_c = \frac{(As+An)}{2} + \frac{(As-An)}{2} \cos(2\gamma), \quad \text{normal slip: } -180^\circ < \gamma < 0^\circ \quad (4b)
$$
A methodology for assessing directivity-centering of GMPE data was presented. The method was used to test directivity centering of NGA-West2 near-source data and the NGA-West2 GMPEs. Four were found to be nearly directivity-centered for spectral periods below 5 seconds. It was briefly explained how the method can be used for record sampling to create directivity-unbiased data for developing GMPEs. Using NGA-West2 finite fault information and a number of test cases, impacts of various parameters on the directivity centering term in the directivity model developed by the author were investigated and relations for the centering term $\xi_c$ were obtained.

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**References**