IMPROVED APPROACH TO ESTIMATE THE PROBABILITY OF COLLAPSE OF STRUCTURES DURING EARTHQUAKES

E. Miranda\textsuperscript{1} and H. Dávalos\textsuperscript{2}

ABSTRACT

The objective of this paper is to summarize an improved approach to estimate the probability of collapse. The proposed approach relies, on one hand, on the use of improved parameters to describe the ground motion intensity, and on the other, on improved methodologies that can reduce the computational effort while at the same time improve the accuracy of the results. It is shown that the proposed intensity measures are significantly better correlated with collapse than currently used parameters and therefore lead to much smaller dispersion of collapse intensities, thus requiring one third or less than the number of ground motions required with traditional intensity measures to achieve a certain level of accuracy. Results are presented showing that estimating the probability of collapse using incremental dynamic analyses in which a model of the structure is subjected to ground motions scaled at increasing levels of intensity until collapse is produced may introduce significant bias in the results. An improved procedure, referred to as Enhanced Two Stripe Analysis, E2SA is presented which provides not only more accurate results compared to those obtained with an incremental dynamic analysis but can be obtained at a small fraction of the computational effort. Extensive studies conducted by the authors over a wide range of structures indicate that new intensity measures combined with the E2SA provide significantly better results, that are unbiased, more accurate and required a smaller computational effort.

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Improved approach to estimate the probability of collapse of structures during earthquakes

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ABSTRACT

The objective of this paper is to summarize an improved approach to estimate the probability of collapse. The proposed approach relies, on one hand, on the use of improved parameters to describe the ground motion intensity, and on the other, on improved methodologies that can reduce the computational effort while at the same time improve the accuracy of the results. It is shown that the proposed intensity measures are significantly better correlated with collapse than currently used parameters and therefore lead to much smaller dispersion of collapse intensities, thus requiring one third or less than the number of ground motions required with traditional intensity measures to achieve a certain level of accuracy. Results are presented showing that estimating the probability of collapse using incremental dynamic analyses in which a model of the structure is subjected to ground motions scaled at increasing levels of intensity until collapse is produced may introduce significant bias in the results. An improved procedure, referred to as Enhanced Two Stripe Analysis, E2SA is presented which provides not only more accurate results compared to those obtained with an incremental dynamic analysis but can be obtained at a small fraction of the computational effort. Extensive studies conducted by the authors over a wide range of structures indicate that new intensity measures combined with the E2SA provide significantly better results, that are unbiased, more accurate and required a smaller computational effort.

Introduction

Even though the most important, and often the only, objective in earthquake resistant design is to prevent the collapse of the structure, the probability of collapse is rarely explicitly computed. This is true for both the design of new structures as well as for the evaluation of existing structures. Recent developments of advanced nonlinear analytical models that are capable of explicitly incorporating the effects of degrading phenomena in materials, structural members and

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connections such as yielding, hardening, softening, local and global buckling, crushing, fracture, bond slippage, cyclic degradation etc. together with much larger computational power in modern multi-core parallel processors now allow the engineers to conduct response history analyses that would have been impossible just a decade ago. Nevertheless, assessing the probability of collapse of structures remains a technically challenging and computationally demanding task for earthquake engineers because of practical guidelines and methods that can provide relatively accurate results while at the same time reduce the computational effort involved. The latter is particularly important when large models are involved.

Villarverde [1] recently reviewed various methods to assess the seismic collapse capacity of building structures. Ibarra and Krawinkler [2] analyzed SDOF and MDOF models that combined $P$-$\Delta$ effects and a new hysteretic model which incorporated various kinds of strength and stiffness deterioration. In particular, that study examined the effect of both record-to-record variability and modeling uncertainty on collapse risk and developed a rational procedure to compute collapse fragility curves and to combine them with the seismic hazard curve at the site to compute the mean annual frequency of collapse. In their procedure, the collapse fragility function was obtained by conducting incremental dynamic analyses [3] of the model of the structure by subjecting the structure to a set of ground motions in which each ground motion is scaled at increasing levels of intensity until the model of the structure collapses. Their procedure was subsequently used in several studies at Stanford’s John A. Blume earthquake Engineering Center aimed at assessing the collapse of various types of structures. While the approach proposed by Ibarra and Krawinkler to estimate the mean annual frequency of exceedance is a rational procedure it is computationally demanding because it requires using 30 to 40 ground motions scaled at many increasing levels of intensity. Furthermore, the procedure requires the determination of the specific value of the ground motion intensity that produces the collapse of the structure. This typically requires combining root searching numerical analyses techniques with more nonlinear response history analyses (NRHA) requiring many hundreds and in some cases thousands of NRHAs.

Some studies have suggested that a structure is deemed to have an acceptable level of safety against collapse if it has a probability of collapse equal or smaller than 10% in the maximum considered earthquake (MCE) [4,5]. The latter is referred to an acceptable conditional probability of collapse because the acceptable probability of collapse is evaluated conditioned on the occurrence of MCE ground motion levels. More recently, the FEMA P-695 document [6] defined an acceptable probability of collapse in terms of the collapse margin ratio defined as the ratio of the median collapse capacity to the MCE intensity level.

The three aforementioned procedures to estimate if a structure have acceptable levels of safety against collapse require the estimation of the collapse fragility function which is the most demanding computational effort in such analysis. The purpose of this paper is to summarize recent studies conducted by the authors aimed at developing improved more reliable methods to assess the probability of collapse of structures subjected to earthquake ground motions. In particular, this paper summarizes an improved and efficient method of assessing the mean annual frequency of collapse referred to as *Enhanced 2 Stripe Analysis, E2SA*. The improved procedure includes three key aspects: (1) Use of improved parameters to quantify the ground motion intensity; (2) Use of better metrics to quantify the risk of collapse; and (3) methods to reduce the computational involved in estimating the probability of collapse.
Improved Collapse Risk Metrics

There have been several metrics that have been proposed in recent years to measure the level of safety against collapse of structures subjected to earthquakes. For example, some studies have proposed the use of median collapse capacity, defined as the spectral acceleration causing the collapse of the model of the structure in 50% of the ground motions considered in the analysis. This provides a measure of safety of collapse for structures with the same fundamental period of vibration located at the same site but not for structures with different periods in the same site nor for structures with the same period at sites with different levels of seismic hazard. Furthermore, it does not take into difference in conditional probability of collapse at other levels of ground motion intensity and therefore it is not recommended to be used as a metric of collapse risk.

Another commonly used option is the conditional probability of collapse of the MCE ground motion intensity. For example, ASCE 7 since its 2010 version [4,5] or the FEMA P695 [6] consider a limit of 10% probability of collapse at the MCE level as an acceptable level of safety against collapse. This metric is much better than just using the median collapse capacity as a measure of safety against collapse because it accounts for some information about the collapse fragility of the structure and of the seismic hazard curve at the site. Since the spectral acceleration at MCE is period dependent it considers the difference in seismic hazard for structures with different fundamental periods of vibration and differences in seismic hazard from one site to another. It is still deficient in the sense that only characterizes the collapse fragility of the structure at a single level of ground motion intensity failing to account for differences in probability of collapse at lower or higher levels of intensity. Similarly, it only characterizes the seismic hazard at the site with one point of the seismic hazard curve at the site.

In the FEMA P695 methodology the collapse risk is evaluated in terms of the acceptability of a calculated collapse margin ratio, which is the ratio of the ground motion intensity that causes median collapse, to the MCE ground motion intensity defined by the building code [6]. Acceptability is measured by comparing the collapse margin ratio, after some adjustment, to acceptable values that depend on the quality of information used to define the system, total system uncertainty, and established limits on acceptable probabilities of collapse (10% at the MCE level). To account for unique characteristics of extreme ground motions that lead to building collapse, the collapse margin ratio is converted to an adjusted collapse margin ratio. Although, again this metric is better than just using the median collapse capacity it fails to to account for differences in probability of collapse at lower or higher levels of intensity than the median collapse capacity. Furthermore, it places emphasis on the middle point of the collapse capacity, while several studies that have conducted collapse deaggregation have indicated ground motion intensities primarily contributing to collapse risk are located in the lower half of the collapse fragility function even for sites with relatively low slopes in the seismic hazard curves [7-9].

The best metric for measuring the level of safety against collapse is the mean annual frequency (MAF) of collapse or the probability of collapse over another time span such as 50 years. This metric of collapse properly accounts for the probability of collapse at all levels of ground motion intensity, that is, considers a full fragility function, and it also considers the seismic hazard at the site by its seismic hazard curve and not by a single ground motion intensity level. The proposed approach makes use of this improved collapse risk metric.
Improved Ground Motion Intensity Measures

The measure of intensity of a ground intensity most commonly used in recent years is the five percent damped spectral acceleration ordinate at the fundamental period of the structure, $\text{Sa}(T_1)$, because it is the same ground motion intensity used in probabilistic seismic hazard analyses and more recently also used in various research projects. While $\text{Sa}(T_1)$ provides an exact measure of intensity of the peak deformation of an elastic SDOF system, its efficiency to estimate seismic behavior of structures rapidly diminishes with increasing level of nonlinearity and it leads to large record-to-record variability when used to estimate large nonlinear deformations in MDOF structures. Figure 1 shows the spectral acceleration $\text{Sa}(T_1)$ by which 274 earthquake ground motions need to be scaled to in order to produce the collapse of a post-Northridge 4-story steel moment resisting steel building [8,9]. The ground motions used in that study were selected based of a seismic hazard deaggregation at the site and had moment magnitudes between 6.9 and 7.6 and Joyner-Boore distances (horizontal distance between the site and the projection of the fault rupture onto the surface) between 0 and 27 km and on sites classified as NEHRP site classes C or D. It can be seen that the ground motions intensities, when characterized by $\text{Sa}(T_1)$, exhibit a very large record-to-record variability with some ground motions producing the collapse of the structure when the record is scaled to a spectral ordinate of 0.48g at $T_1=1.33s$ while others need to be scaled to spectral ordinates as large as 3.27g (almost seven times larger) to produce the collapse of the structure. The median collapse intensity which for this structure is 1.03g, and the 95 percentile (2.11g) is 3.64 times larger than the intensity corresponding to the 5 percentile (0.58g) indicating a large variability of the ground motion intensity. The corresponding logarithmic standard deviation is 0.39 which is very large.

Although originally developed as a tool to establish seismic performance factors for generic seismic-force-resisting systems, the FEMA P695 methodology was adapted in its Appendix F [6] for collapse assessment of individual buildings. That methodology includes an approximate method to take into account the spectral shape of the records in which the median collapse capacity of the collapse margin ratio is adjusted by considering the parameter epsilon, $\varepsilon$,
which is defined as the number of logarithmic standard deviations between the observed spectral value and the median prediction from an attenuation function. The $\varepsilon$ value that is a function of the ground motion record, the period of interest, and the attenuation function used for prediction is then used to computed an adjusted collapse by using a linear relationship given by: $\ln[S_{a\text{, col}}(T_1)] = \beta_0 + \beta_1 \varepsilon$. Where the value of $\beta_0$ indicates the average collapse capacity when $\varepsilon = 0$, and the value of $\beta_1$ indicates how sensitive the collapse capacity $S_{a\text{, col}}$ is to changes in the $\varepsilon$ value.

The figure in the middle of figure 1 shows the adjusted collapse capacities using the approach used in the FEMA P695 methodology. However, rather than using generic coefficients $\beta_0$ and of $\beta_1$, here coefficients specific to the structure and ground motions were obtained by conducting a linear regression analyses in order to apply the best possible correction according to that approach by increasing or reducing the collapse intensity depending on the value of $\varepsilon$ of each record. As shown in this figure, the record-to-record variability remains very large because, contrary to observations from some previous studies, $\varepsilon$ is not a good measure of spectral shape as it only contains information at a single period [9,10]. For this structure, adjusting collapse capacities reduces the logarithmic standard deviation only from 0.39 to 0.37 which is only a 5% reduction in dispersion.

Recently Eads et al. [9,11] proposed a new intensity measure, $S_{a\text{avg}}$, consisting on the geometric mean of spectral ordinates between 0.2 times the fundamental period of vibration to three times the fundamental period of vibration. This intensity measure differs from the one used by Bianchini [12] in two ways: (1) It uses a much wider range of periods to computed the averaged spectral ordinate; and (2) uses not only spectral ordinates of periods longer than the fundamental period of vibration but also periods shorter than the fundamental period of vibration. Including spectral ordinates of periods shorter than the fundamental period is important not so much as to capture demands on higher modes of the structure as some studies have suggested, but rather because it indirectly captures information about critical pulses in the ground motion. The best indication that this is the case is when comparing the reduction in logarithmic deviation in collapse intensities even in single-degree-of-freedom systems, which of course, by definition do not have higher modes, yet important reductions in dispersion are still achieved [9-11].

Figure 1 also shows the record-to-record variability of the collapse intensities of the 274 ground motions but now considering $S_{a\text{avg}}$, as the intensity measure. A Large reduction in dispersion is apparent with a reduction in logarithmic standard deviation from 0.39 when using the conventional $S_a(T_1)$ to 0.22, a 44% reduction when using this new intensity measure. It is important to recall that the standard error is directly proportional to the standard deviation of the estimate and inversely proportional to the square root of the sample size, which means that using this intensity measure, for the same desired level of standard error it is possible to reduce the number of ground motions to approximately one third of those required when using $S_a(T_1)$ or $S_a(T_j)$ adjusted with $\varepsilon$. As explained in [9,10] $S_{a\text{avg}}$, is a significantly better intensity measure because it contains information about the ground motion intensity at a wide range of frequencies/periods, but also because it inherently contain information about $SaRatio$, the ratio of spectral ordinates at the fundamental period of vibration of the structure to the geometric mean of spectral ordinates to the left and to the right of the fundamental period and hence it provide a measure of how high the spectral ordinate at the fundamental period of vibration is relative to neighboring spectral ordinates thus providing information of the spectral shape of each ground motion.
Eads et al. [9,11] conducted a comprehensive evaluation of $S_{a_{avg}}$ including studies on ad hoc selection of period range for each structure, the number and spacing of periods, whether geometric mean or average spectral ordinate was better and evaluated this parameter on almost 700 different buildings subjected to different ground motion sets. The reader is referred to those studies for further details.

Even though $S_{a_{avg}}$ provides a significantly better ground motion intensity measure than the commonly used $S_{a(T_i)}$ or the adjusted $S_{a(T_i)}$ considering the parameter $\varepsilon$, the authors have continued to study and search of improved ground motion intensity measures. Figure 2, compares the three previously mentioned intensity measures to a new IM, recently proposed by the authors referred to as $FIV3$ [13].

**Improved Methods to Estimate MAF of Collapse**

In ASCE 7-16 chapter 12, the performance is judged not to be acceptable when one or two ground motions out of 11 failed to converge or exceed acceptable criteria. Computing the conditional probability of non-compliance as the fraction of non-compliance out of a small sample such as 7 or 11 ground motions is unreliable. As shown by Eads et al. [8] such probability corresponds to a binomial probability distribution. Furthermore, they showed that the level un uncertainty in such fraction to collapse or non-converging ground motions to the total number of ground motions is strongly influenced by the sample size and that it is an unreliable procedure when used in conjunction with small sample sizes such as 7 or 11 ground motions. The actual probability of collapse could be much larger or much smaller than the one determined with such procedure, hence the authors do not recommend its use.

In most previous studies that have obtained an estimate of the collapse fragility function, which described the variation of the probability of failure as a function of a ground motion intensity, the fragility function has been obtained by using incremental dynamic analyses [3] in which each ground motion was scaled at increasing levels of intensity until it produced the collapse.
of the model of the structure. Collapse intensities of each ground motion were then sorted and their rank was normalized by the number of ground motions to estimate the conditional probability of collapse or by using another “plotting position”. Although these procedure has been used in many previous studies, it is very computationally expensive. In particular, determined the exact intensity measure at which a ground motion triggers the collapse of the model required a large computational effort as it requires iteration under many scaling factors until the collapsing intensity is determined. More importantly, as shown in [8] this is not required, and they proposed a far more efficient method in which nonlinear response history analyses are carried out only at two different intensity levels and a lognormal probability distribution is fitted through them, which simply requires the solution of two equations with two unknowns to obtain reliable estimates of the collapse fragility function.

The authors conducted additional studies to improve the method developed by Eads et al. [8]. In the improved method, referred to as Enhanced 2 Stripe Analysis, E2SA, the following recommendations are made: (1) It is recommended that the two levels of intensities in which response history analyses are conducted be determined using a rough a-priori estimate of the collapse fragility function together with a collapse deaggregation analysis [7] which take into account the seismic hazard curve at the site rather than selecting then at fixed conditional probabilities of collapse; (2) it is recommended to use different set of ground motions at each stripe and to avoid scaling factors larger than two as larger scaling factors may introduce considerable bias in the results [14]; (3) it is recommended to first conduct analyses at the higher level of intensity and to use those results to refine the intensity level in which the response history analyses will be conducted for the lower stripe; and (4) improved results are obtained by using twice the number of ground motions in the lower stripe than in the upper stripe as the lower half of the collapse fragility function has a larger contribution to the mean annual frequency of collapse. Figure 3 on the left shows the mean ratio as a measure of the bias (error) with respect to the “true” MAF of collapse at three different sites using different proportions of number of ground motions in the higher stripe, NH, with respect to the number of ground motions in the lower stripe NL. The figure on the right shows the width of the 95% confidence intervals of the estimates of the MAF of collapse. It is shown that using NH/NL of approximately 0.5, that is, using twice as many ground motions in the lower stripe than in the upper stripe as the lower half of the collapse fragility function has a larger contribution to the mean annual frequency of collapse. For further details and examples of application of the E2SA, the reader is referred to [15].

Figure 3. Bias and variability of estimates of MAF of collapse as a function of the ratios of the number of analyses conducted in the higher (H) and lower (L) intensity stripes.
Summary and Conclusions

Avoidance of collapse is the most important objective of earthquake resistance design. Notwithstanding the importance of avoiding collapse, current building codes do not include methods to verify that the structure has adequate safety against collapse. Computing the mean annual frequency of collapse or the probability of collapse during the life of the structure is a technically challenging and computationally demanding task. It requires having adequate criteria to select ground motions and as which measures of ground motion intensity to use, knowing how many records to use, the number and location of intensity levels at which to conduct nonlinear response history analyses as well as the best metrics to use for measuring the level of safety of a structure against collapse.

It has been shown that collapse risk metrics based on the median collapse intensity or on the probability of collapse at a single level of intensity such as the probability of collapse at the MCE level are not good metrics as they do not include information about the seismic hazard at the site or incomplete information about the probability of collapse at various levels of ground motion intensity and better representation of the seismic hazard at the site. The use of the mean annual frequency of collapse or the probability of collapse over a period of time much as probability of collapse in 50 years provide significantly better measures of collapse risk.

Results have been presented that show that selecting records based on $\varepsilon$ or using this parameter to correct collapse intensities while it may reduce some of the bias in the results still leads to large record-to-record variabilities which are not much better than the one obtained using the spectral ordinate at the fundamental period as the ground motion intensity measure. This is because, $\varepsilon$ is not a good measure of spectral shape and therefore fails to reduce record-to-record variability. Furthermore, selection of records only based on $\varepsilon$ and then increasing their intensity as done in the FEMA P695 methodology can bias the results. Meanwhile using improved IMs such as $Sa_{avg}$ or $FIV3$ leads to systematic large reductions in variability while at the same time producing more reliable and more consistent estimates of the mean annual frequency of collapse for different ground motion sets.

An improved method, referred to as Enhanced 2 Stripe Analysis, E2SA, has been presented and results compared to other methods have been summarized. This method provides several improvements with respect to a previous method also based on conducting response history analyses only at two level of intensity. Results are particularly promising as they indicate that it produces reliable estimates of the mean annual frequency of collapse while at the same time reduces the computational effort respect to several existing methods.

Acknowledgments

This study is a follow up investigation to one conducted by the senior author with a former PhD student, Dr. Laura Eads and Prof. Dimitrios Lignos. Their contributions to the early stages on this investigation are greatly acknowledged. The second author acknowledges financial support from Consejo Nacional de Ciencia y Tecnología (CONACYT) in Mexico and the John A. Blume Fellowship to pursue his doctoral studies at Stanford University under the supervision of the first author.
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