CAPACITY-BASED INELASTIC DISPLACEMENT SPECTRUM FOR REINFORCED CONCRETE BRIDGE COLUMNS

Ping-Hsiung Wang¹, Yu-Chen Ou², and Kuo-Chun Chang³

ABSTRACT

This paper aims to construct inelastic displacement spectra associated with corresponding damage state (i.e., the capacity-based inelastic displacement spectrum) for RC bridge columns using a new smooth hysteresis model proposed by the authors. Nonlinear time history analysis of SDOF systems having different hysteretic behaviors identified from the tested columns with various design parameters and subjected to 15 far-field and 15 near-fault pulse-like ground motions was conducted. The considered design parameters include the ratios of longitudinal reinforcement and transverse reinforcement, aspect ratio, as well as axial load ratio of columns, and the effects of these parameters on deteriorated hysteretic behaviors of RC bridge columns was discussed. Failure criteria was made because columns analyzed using the model proposed by the authors could fail under one or more than one of the earthquake records especially when subjected to near-fault ground motions. Furthermore, the effects of various design parameters of column and earthquake types on the computed spectra were presented and the well-known inelastic displacement ratio formula in the literatures were evaluated using the computed spectra. Finally, formulae for inelastic displacement ratio and corresponding damage state as a function of structural period, strength ratio, and various design parameters were established for far-field and near-fault ground motions, respectively.

¹ Associate Technologist, National Center for Research on Earthquake Engineering, Taipei, Taiwan (email: phwang@ncree.narl.org.tw)
² Professor, Dept. of Civil Engineering, National Taiwan University, Taipei, Taiwan
³ Professor, Dept. of Civil Engineering, National Taiwan University, Taipei, Taiwan
Capacity-Based Inelastic Displacement Spectrum for Reinforced Concrete Bridge Columns

Ping-Hsiung Wang\textsuperscript{1}, Yu-Chen Ou\textsuperscript{2}, and Kuo-Chun Chang\textsuperscript{3}

ABSTRACT

This paper aims to construct inelastic displacement spectra associated with corresponding damage state (i.e., the capacity-based inelastic displacement spectrum) for RC bridge columns using a new smooth hysteresis model proposed by the authors. Nonlinear time history analysis of SDOF systems having different hysteretic behaviors identified from the tested columns with various design parameters and subjected to 15 far-field and 15 near-fault pulse-like ground motions was conducted. The considered design parameters include the ratios of longitudinal reinforcement and transverse reinforcement, aspect ratio, as well as axial load ratio of columns, and the effects of these parameters on deteriorated hysteretic behaviors of RC bridge columns was discussed. Failure criteria was made because columns analyzed using the model proposed by the authors could fail under one or more than one of the earthquake records especially when subjected to near-fault ground motions. Furthermore, the effects of various design parameters of column and earthquake types on the computed spectra were presented and the well-known inelastic displacement ratio formula in the literatures were evaluated using the computed spectra. Finally, formulae for inelastic displacement ratio and corresponding damage state as a function of structural period, strength ratio, and various design parameters were established for far-field and near-fault ground motions, respectively.

Introduction

To evaluate the seismic performance of new designed or existing bridges, various analysis methods are available depending on the seismicity, regularity, complexity, and importance of bridges. Among the current practices in approximate seismic evaluation [1-5], there is a trend towards using the well-known displacement coefficient method [6].

Many researchers had focused their efforts on the relationship between maximum inelastic and maximum elastic displacement for systems having the same period of vibration (referred to as inelastic displacement ratio, hereafter). Most, if not all, of the inelastic displacement ratios computed in the literatures are based on polygonal hysteretic models, such as the elastoplastic model, the modified-Clough model, the Takeda model, and some stiffness and strength degrading models. However, real hysteresis behaviors of structures are smooth rather than piecewise linear

\textsuperscript{1} Associate Technologist, National Center for Research on Earthquake Engineering, Taipei, Taiwan (email: phwang@ncree.narl.org.tw)

\textsuperscript{2} Professor, Dept. of Civil Engineering, National Taiwan University, Taipei, Taiwan

\textsuperscript{3} Professor, Dept. of Civil Engineering, National Taiwan University, Taipei, Taiwan
with abrupt stiffness changes. Furthermore, the severity and pattern of degradation of hysteresis behaviors depend mainly on the structural system and design parameters of a structure, thus influencing the resulting seismic responses. But current published formulae for inelastic displacement ratio evidently lack this kind of linking. For bridge application, there should be a better chance to fill this gap since the seismic behavior of a bridge is primarily dominated by its substructure (i.e., bridge pier or column), which is much simpler than a building. Consequently, there is a need to compute inelastic displacement ratio using a smooth hysteretic model capable of reflecting the corresponding hysteresis behavior to its design parameters. The authors [7] recently proposed a versatile smooth hysteretic model that can realistically simulate the stiffness degradation, strength deterioration, pinching behavior, and various characteristics regarding the path dependence of reinforced concrete (RC) bridge columns, among which the damage index proposed by Park and Ang [8] was correlated with strength deterioration and was proved to be a good indicator for damage assessment of column regardless of imposed loading history. Accordingly, nonlinear time history analysis via the model can evaluate not only the required maximum response of column but also its damage state after a given ground motion, giving a better insight into its seismic performance other than merely a structural demand.

The objective of this paper is to statistically compute the inelastic displacement ratio associated with corresponding damage state (i.e., the capacity-based inelastic displacement spectrum) of SDOF system with constant relative strength using a versatile smooth hysteretic model that can take into account the effects of various design parameters of RC bridge columns on deteriorated hysteresis behaviors when subjected to both far-field and near-fault ground motions. The computed data was then used to evaluate the well-established formulae for inelastic displacement ratio and new formulae for the capacity-based inelastic displacement spectrum as a function of relative strength, period of vibration, and various design parameters of RC bridge columns were proposed for far-field and near-fault ground motions, respectively.

**Smooth Hysteretic Model**

The smooth hysteretic model used to perform the nonlinear time history analysis of SDOF system in this paper is proposed by Wang et al. [7]. The model is based on the well-known Bouc-Wen model [9] with significant modification to consider hysteretic rules for damage accumulation and path dependence of RC bridge columns in which relationship between the restoring force and displacement of system is described by a differential equation as

\[
\dot{z}(t) = h(z) \left( A \dot{u} - \nu \dot{u} \|z\|^{n-1} z + \gamma \dot{u} \|z\|^n \right)
\]

\[
h(z) = 1 - \xi_1 \exp \left\{ - \frac{[z \text{sgn} (\dot{u}) - q \ddot{u} |\ddot{u}|^2]}{\xi_2^2} \right\} \left| \frac{\text{sgn} (\dot{u}) + \text{sgn} (z)}{z} \right|
\]

where \(z\) and \(u\) are the normalized restoring force and displacement of system, respectively; \(A, \beta, \gamma, \text{ and } n\) are parameters that determine the basic shape of the \(z-u\) relation; \(\nu\) and \(\eta\) are the strength and stiffness degradation parameters, respectively; and \(h(z)\) is the pinching function in which \(\xi_1\) controls the severity of pinching, \(\xi_2\) controls the spread of pinching, and \(q\) defines the force level \(z\) at which pinching reaches its maximum effect.

Equation (1) defines essentially the continuous change of stiffness of hysteresis loop with respect to its force level. Equation (1) defines essentially the continuous change of stiffness of hysteresis loop with respect to its force level \(z\). Figure 1(a) plots in the \(z-u\) plane the hysteresis
loop of the 4% drift of column COC conducted by Wang et al. [7], and Figure 1(b) illustrates in the $dz/du$-$z$ plane the corresponding measured and simulated stiffness variation for the reloading curve in the negative direction. It can be clearly seen that reloading behavior is a combined effect of stiffness degradation and pinching on which each model parameter plays a specific role. For strength deterioration, the model utilizes the damage index ($DI$ as defined in Eq. 3) proposed by Park and Ang [8] to initiate strength deterioration until $DI$ is accumulated to a threshold value $DI_o$ that can be identified from experimental results. More details regarding the model calibration and hysteretic rules for path dependence can be found in Wang et al. [7].

$$DI = \frac{\delta_m}{\delta_u} + \lambda \int \frac{dE}{F_y \delta_u}$$  \hspace{1cm} (3)

where $\delta_m$ is the maximum displacement of column; $\delta_u$ is the ultimate displacement capacity of column under monotonic loading; $\int dE$ is the accumulated hysteretic energy dissipation; $F_y$ is the yield force; and $\lambda$ is a parameter to correlate hysteretic energy dissipation to damage, which can be calculated by setting $DI$ equal to one at the ultimate state of column when the strength of column drops to 80% of its peak value.

![Figure 1. (a) Hysteresis loop of 4% drift of column COC and (b) corresponding $dz/du$-$z$ relation for reloading in the negative direction](image)

**Model Identification for RC Bridge Columns with Various Design Parameters**

In order to consider the effects of design parameters of RC bridge columns on deteriorated hysteretic behaviors and thus on computed capacity-based inelastic displacement spectrum, model identification from relevant experimental data was conducted. The design parameters of RC bridge column considered include the longitudinal and transverse reinforcement ratios, the aspect ratio, and the axial load ratio. Test data focusing on these effects are mainly obtained from the structural performance database of Pacific Earthquake Engineering Research Center, PEER [10]. For each of the tested columns, model parameters capable of representing specific degradation characteristic as illustrated in Fig. 1(b) were identified using the methodology proposed by Wang et al. [7], which allowed the effects of certain design parameter to be quantitatively evaluated. Table 1 shows the columns used and corresponding design parameters. Figure 2 shows good agreement between the experimental and simulated hysteresis loops, and thus the identified model parameters are deemed suitable to represent their degradation characteristics. The identified model parameters regarding stiffness degradation ($\eta$) and pinching behavior ($q, \xi_1, \text{and } \xi_2$) with the effects of various design parameters are shown in Figure 3, where more detailed discussions can be found in Wang [11].
Figure 2. Experimental and simulated hysteresis loops for columns with various design parameters.

Figure 3. Identified model parameters considering the effects of: (1) longitudinal reinforcement; (2) transverse reinforcement; (1) aspect ratio; and (4) axial load.
Table 1. Design parameters of columns

<table>
<thead>
<tr>
<th>Hysteretic Model</th>
<th>M1(EPP)</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
<th>M9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen Name</td>
<td>-</td>
<td>1015</td>
<td>415</td>
<td>407</td>
<td>430</td>
<td>COC/CLC</td>
<td>CTR1</td>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>Long. Reinf. Ratio (%)</td>
<td>1.5</td>
<td>1.5</td>
<td>0.75</td>
<td>3.0</td>
<td>2.1</td>
<td>2.1</td>
<td>2.2</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Trans. Reinf. Ratio (%)</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>1.26</td>
<td>1.79</td>
<td>0.94</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>-</td>
<td>10</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>3.2</td>
<td>3.5</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>Axial Load Ratio ((f_c^2/A_g))</td>
<td>-</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.10</td>
<td>0.10</td>
<td>0.09</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Capacity-Based Inelastic Displacement Spectrum

The proposed spectrum is composed of an inelastic displacement spectrum associated with corresponding damage states of RC bridge columns that have various design parameters and subjected to far-field and near-fault pulse-like ground motions. Failure criteria was made because columns analyzed by the proposed model could fail under one or more than one of the earthquake excitations especially when subjected to near-fault ground motions. The effects of design parameters of column and earthquake types on the calculated spectra were presented and then used to evaluate the well-known displacement spectra in the literatures. Finally, formulae for inelastic displacement ratio and corresponding damage state as a function of structural period, strength ratio, and design parameters were established for far-field and near-fault ground motions by means of nonlinear regression.

Spectrum Parameters

To estimate the maximum inelastic displacement demand of a structure having constant lateral strength and subjected to ground excitation, the inelastic displacement ratio \( C_R \) was introduced for simplified analysis procedures as implemented in FEMA 273 [6]. And the lateral strength of the inelastic system is described by lateral strength ratio \( R \) (or strength reduction factor).

\[
C_R = \frac{\Delta_{\text{inelastic}}}{\Delta_{\text{elastic}}} \quad (4)
\]

\[
R = \frac{mS_a}{f_y} \quad (5)
\]

where \( m \) is the mass of the system, and \( S_a \) is the elastic spectral acceleration, the product of which represents the lateral strength needed to maintain the system elastic.

In this research, inelastic displacement ratios were calculated for SDOF systems with a viscous damping ratio of 5%, having 9 hysteretic behaviors (denoted as M) and 5 lateral strength levels (denoted as R), and subjected to 15 far-field (denoted as FF) and 15 near-fault ground motions (denoted as NF). The ground motion records used can be found in [11] where a wide range of pulse period content ranging from 1 s to 10 s was included in the selected near-fault records. Moreover, the lateral strength ratios \( R=1.5, 2, 3, 4, \) and 5 were considered. For each hysteretic behavior, each lateral strength ratio, and each ground motion, the inelastic displacement ratios were computed for a set of 20 periods of vibration between 0.05 and 3.0 s. In addition, the corresponding damage indexes DI were also computed to indicate the damage state at the end of
each ground excitation. As a result, each inelastic displacement spectrum would be associated with a corresponding damage state spectrum to form a twin spectrum for each combination of spectrum parameters.

**Failure Criteria**

As evidenced in [7], the damage index proposed by Park and Ang [8] can be a good index to represent the damage state of a RC bridge columns under hysteresis in terms of visual damage condition and strength deterioration. The parameters of damage index $DI$ were calibrated based on the assumption that $DI = 1.0$ corresponds to the lateral strength of column dropping to 80% of its peak lateral strength. Accordingly, when performing the nonlinear time history analysis for $C_R$ spectrum, a column is deemed failure or collapse if its damage index ($DI$) accumulated under ground excitation is larger than one. On the other hand, in constructing the $C_R$ and $DI$ spectra, the mean values of $C_R$ and $DI$ over 15 far-field or near-fault ground motions were computed for each combination of spectrum parameters (i.e., each structural period, lateral strength ratio, and hysteretic behavior). However, as mentioned previously columns analyzed by the proposed model could fail under one or more than one of the ground motions especially when subjected to near-fault pulse-like records. If we exclude the cases identified as failure from the 15 cases and take the mean value of the remaining ones, the results could be distortive as the number of exclusion increase. Consequently, an assumption was made such that if the probability of failure is larger than 20% among 15 far-field or near-fault ground motions, columns devised using that set of parameters will be deemed unable to survive under considered earthquakes.

**Selected Analyzed Results**

Selected results for systems having different longitudinal reinforcement ratios (i.e., 0.75% and 3.0% for M4 and M5 models, respectively) and subjected to two types of earthquakes (i.e., far-field and near-fault ground motions) were demonstrated. Fig. 4 shows the calculated $C_R$ and $DI$ spectra. It can be found that the calculated spectra differed dramatically for systems subjected to far-field and near-fault ground motions. More detailed discussions can be found in Wang [11].

![Figure 4](image-url)
Evaluation of Well-Known $C_R$ Formulae

To evaluate the accuracy of the well-known $C_R$ formulae in the literatures, it was assumed the model used is representative of real hysteresis behavior of RC bridge columns and the $C_R$ spectrum computed using the M6 model was used to be a benchmark. Figures 5(a1)-(d1) show comparisons between the $C_R$ spectra provided by FEMA 273 [6], FEMA 440 [12], Ruiz-Garcia and Miranda [13], and FHWA [1] (or Eurocode 8 [5]) with the computed $C_R$ spectra for far-field ground motions, respectively. And Figures 5(a2)-(d2) show comparison results for near-fault ground motions. Since the $C_R$ spectra analyzed in this research did not consider the P-Delta effects, the P-Delta effects were included in the analysis. Furthermore, the characteristic period, $T_c$ used was taken as 0.55 that was recommended by FEMA 440 [12] for NEHRP site class C, considering most of the ground motion records belonged to this category. The constant coefficients $a$, $b$, $c$, and $d$ in Ruiz-Garcia and Miranda [13] are 48, 1.8, 50, and 0.85, respectively, as recommended for site class C.

Figure 5. Evaluation of well-known $C_R$ formulae using computed $C_R$ spectra for (1) far-field; and (2) near-fault ground motions

Figure 5(a1) shows the $C_R$ spectrum from FEMA 273 is practically independent of $R$ due to a capping imposed on $C_1$ and the equal-displacement rule does not hold for periods larger than around 0.8 s, resulting in overestimation of $C_R$ compared to that computed by M6 model. Figure 5(b1) shows the $C_R$ spectrum from FEMA 440 underestimates $C_R$ for periods smaller than around 0.8 s, while a similar trend can be found from the $C_R$ spectrum of Ruiz-Garcia and Miranda shown in Figure 5(c1). Figure 5(d1) shows surprisingly good agreement between the $C_R$ spectrum of FHWA (or Eurocode 8) and the computed $C_R$ spectrum by M6 model for far-field ground motion. It should be reminded that the $C_R$ formula of FHWA is identical to the $C_1$ formula of FEMA 273 except that a capping over $C_1$ was excluded and that $T_c$ was replaced by $1.25T_c$ after which the equal-displacement rule applied. The $C_1$ in FEMA 273 was intended to relate the expected maximum displacement of inelastic SDOF system with EPP hysteretic behavior to that of corresponding elastic system. In fact, however, it stemmed from the formula for strength reduction...
factor $R_\mu$ proposed by Vidic et al. [14] using a stiffness degrading Q-model, which was then transformed by Fajfar [15] into the form of $C_R$ (or $C_1$). Notwithstanding this kind of derivation would cause statistical bias and underestimation of inelastic displacement ratio compared to those computed directly from nonlinear response analysis [16], the $C_1$ formula still coincides well with the computed results. On the contrary, all these well-known formula obviously underestimate the $C_R$ when computed for near-fault ground motions as shown in Figs. 5(a2)-(d2).

**Proposed $C_R$ and DI formula**

As shown in Fig. 4 the computed $C_R$ spectrum existed a period limit in different design scenarios so that a system having period of vibration less than this limit value would receive a probability of failure larger than 20% and was deemed unable to survive under considered earthquakes. Considering the period limit was identified according to its damage indices, it was then found using the damage index to determine the limit could be a feasible way. This can be illustrated in Figure 6 where the period of vibration corresponding to 1 indicates the limit. However, as presented in the previous DI spectra, the DI corresponding to its period limit did not reach 1 since only 20% cases had been identified as failure. As a result, using this way would cause overestimate the DI in the vicinity of the limit, which was believed to be expedient.

The proposed $C_R$ and DI formula were based on the $C_R$ of FHWA (or Eurocode 8) with some modification. According the calculated spectra for various spectrum parameters, the $C_R$ and DI formula for far-field ground motions were established using nonlinear regression analysis as follows:

$$C_R = \begin{cases} 
1 - \frac{1}{R} \frac{T_e}{T_1} + \frac{1}{R}, & T_e < T_1^* \\
1.0, & T_e \geq T_1^* 
\end{cases}$$

$$R = \begin{cases} 
1.5, & \text{FF} \\
2.0, & \text{NF}
\end{cases}$$

Figure 6. Illustration of using DI spectrum to limit the available spectrum range of corresponding $C_R$ spectrum in (1) far-field cases; and (2) near-fault cases.
\( T_1^* = C_{T1} T_s \)  
(7)

\( C_{T1} = \begin{cases} 1.25, & R = 1.5 \\ 1.45, & R \geq 2.0 \end{cases} \)  
(8)

\[ DI = \begin{cases} \left( \frac{C_{DI} - C_{DI}}{R} \right) \left( \frac{T_2}{T_e} \right)^n + \frac{C_{DI}}{R}, & T_e < T_2^* \\ C_{DI}, & T_e \geq T_2^* \end{cases} \]  
(9)

\( T_2^* = C_{T2} T_s \)  
(10)

\( C_{DI} = [0.061 - 0.009 \ln(AD) + 0.017LR - 0.015 \ln(TR) + 0.032AL]R \)  
(11)

\( C_{T2} = 0.37 \ln(R) + 1.3 \)  
(12)

\( n = 0.1R + 1.1 \)  
(13)

where \( AD \) is the aspect ratio of column; \( LR \) is the percentile value of longitudinal reinforcement ratio; \( TR \) is the ratio of transverse reinforcement to that required by current bridge seismic design codes [17, 18], and \( AL \) is the axial load ratio.

In addition, the \( C_R \) and \( DI \) formula for near-fault ground motions were proposed as follows:

\[ C_R = \begin{cases} \left( \frac{C_{BR} - C_{BR}}{1.5} \right) \left( \frac{T_1}{T_e} \right)^n_1 + \frac{C_{BR}}{1.5}, & T_e < T_1^* \\ C_{BR}, & T_e \geq T_1^* \end{cases} \]  
(14)

\( T_1^* = C_{T1} T_s \)  
(15)

\( C_{T1} = 1.264 \exp(0.187R) \)  
(16)

\( C_{BR} = -0.062R^2 + 0.427R + 0.668 \)  
(17)

\( n_1 = 0.008R^2 + 0.033R + 0.735 \)  
(18)

\[ DI = \begin{cases} \left( \frac{C_{DI} - C_{DI}}{1.5} \right) \left( \frac{T_2}{T_e} \right)^n_2 + \frac{C_{DI}}{1.5}, & T_e < T_2^* \\ C_{DI}, & T_e \geq T_2^* \end{cases} \]  
(19)

\( T_2^* = C_{T2} T_s \)  
(20)

\( C_{T2} = n_2 = C_1 \exp[(1 - C_1)R] + C_2 \exp(-R) \)  
(21)

\( C_{DI} = mR - 0.06 \)  
(22)

\( m = 0.114 - 0.005 \ln(AD) + 0.026LR - 0.025 \ln(TR) - 0.216AL \)  
(23)

\( C_1 = 0.528 + 0.062 \ln(AD) - 0.131LR + 0.171 \ln(TR) + 1.908AL \)  
(24)

\( C_2 = 2.362 - 0.265 \ln(AD) + 0.762LR - 1.141 \ln(TR) - 9.692AL \)  
(25)

Conclusions

Inelastic displacement spectra associated with corresponding damage state (i.e., the capacity-based inelastic displacement spectrum) for RC bridge columns were constructed using a newly proposed hysteresis model. Nonlinear time history analysis of SDOF systems having different hysteretic behaviors identified from the tested columns with various design parameters and subjected to 15 far-field and 15 near-fault pulse-like ground motions was conducted. Finally, formulae for the proposed damage-based inelastic displacement spectra as a function of structural period, strength ratio, and various design parameters were established for far-field and near-fault ground motions, respectively, by means of nonlinear regression analysis.
References

18. MOTC. Seismic Bridge Design Specifications, Ministry of Transportation and Communications (MOTC), Taiwan, 2009. (in Chinese)