EVALUATION OF SHEAR WALL STRUCTURES USING A CONSTANT YIELD DISPLACEMENT PROCEDURE

A. Tsiavos\textsuperscript{1}, B. Stojadinovic\textsuperscript{2}

ABSTRACT

The existing force-based seismic evaluation procedures are based on the simplifying assumption that the vibration period of the structure remains constant and independent of its strength. This assumption leads to small yield displacement estimates and thus, high ductility demand estimates for stiff fixed-base structures. This study shows that such ductility demand values obtained using the constant-period assumption are not realistic.

A new relation between the strength and the deformability of structures is proposed. This relation is based on the argument that the yield displacement of a structure in bending depends only on the yield strain of the yielding material and the geometry of the structure, and that it does not depend strongly on the bending strength of that structure. This new relation between the strength and the deformability of a structure is quantified through a statistical analysis of the response of a fixed-base single-degree-of-freedom inelastic structure subjected to a wide range of recorded ground motions.

This relation lays the foundation for a new performance-based seismic evaluation procedure, the Constant-Yield-Displacement-Evaluation (CYDE) procedure. This procedure is based on the assumption that the strength and the yield displacement of a structure are independent. In comparison to the existing seismic evaluation procedures, it offers a more realistic way to determine the inelastic displacement ductility demand of structures with a predefined strength. The CYDE procedure does not require the vibration period (i.e. the stiffness) of the structure. Instead, this period emerges from the values of the strength and the yield displacement of the structure. The four steps of the CYDE procedure can be easily implemented as they are based on the elements of existing seismic evaluation procedures.

\textsuperscript{1}Postdoctoral Researcher, ETH Zurich, HIT F43, Stefano-Francsini-Platz 5, 8093 Zurich (tsiavos@ibk.baug.ethz.ch)

\textsuperscript{2}Professor, ETH Zurich, HIL E14.1, Stefano-Francsini-Platz 5, 8093 Zurich (stojadinovic@ibk.baug.ethz.ch)
Evaluation of shear wall structures using a constant yield displacement procedure

A. Tsiavos¹, B. Stojadinovic²

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The existing force-based seismic evaluation procedures are based on the simplifying assumption that the vibration period of the structure remains constant and independent of its strength. This assumption leads to small yield displacement estimates and thus, high ductility demand estimates for stiff fixed-base structures. This study shows that such ductility demand values obtained using the constant-period assumption are not realistic.

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Introduction

The relationships between the vibration period $T_n$, the strength reduction factor $R_y$ and the displacement ductility demand $\mu$ for fixed-base structures under earthquake ground motion excitation have been extensively investigated in the past. Newmark and Hall [1], Lai and Biggs [2] and Riddel and Newmark [3] proposed piece-wise linear $R_y-\mu-T_n$ relations for fixed-base structures. Riddel, Hidalgo and Cruz [4] and Vidic, Fajfar and Fischinger [5] presented bilinear approximations for $R_y-\mu-T_n$ relations. Elgadamsi and Mohraz [6], Arias and Hidalgo [7], Nassar and Krawinkler [8], Miranda [9], and Miranda and Bertero [10] suggested continuous nonlinear $R_y-\mu-T_n$ functions. All these relations indicate that the stiff (short-period) fixed-base structures should not be allowed to yield (i.e. $R_y=1$ and $\mu=1$ for short $T_n$). This design requirement is based on the argument that the inelastic seismic displacement ductility demand of yielding stiff structures would be very high.

The inelastic behavior ranges determined in these studies are based on the assumption that the vibration period of the structure remains constant (and equal to the elastic vibration period)

¹Postdoctoral Researcher, ETH Zurich, HIT F43, Stefano-Francisci-Platz 5, 8093 Zurich (tsiavos@ibk.baug.ethz.ch)
²Professor, ETH Zurich, HIL E14.1, Stefano-Francisci-Platz 5, 8093 Zurich (stojadinovic@ibk.baug.ethz.ch)
and independent from its yield strength. According to this assumption, the yield displacement of the structure varies linearly with its strength (Equation 1). For stiff fixed-base structures, this constant-period assumption leads to unrealistically small yield displacements thus inducing unrealistically large displacement ductility demand values.

\[ u_{y,s} = \frac{F_{el,s}}{k_y \cdot R_y} \]  

(1)

These displacement ductility demand values increase even more in the case of base-isolated superstructures, as observed by Sollogoub [11], Vassiliou et al. [12] and Tsiavos et al. [13-15]. This increase is attributed to the small forces acting on the isolated superstructures that, following the constant-period approach, leads to yet smaller yield displacements.

Many researchers (e.g. Priestley [16,17], Aschheim [18,19]) pointed out that the yield displacement of a structure \( u_{y,s} \) is virtually constant given the structure geometry and the choice of the yielding material, and that it not influenced by the change of its strength. This observation is the basis for the displacement-based seismic evaluation procedures.

A new displacement-based seismic evaluation procedure, the Constant Yield Displacement Evaluation (CYDE) procedure, is proposed in this study. This procedure is based on the argument that the yield displacement of a structure is only dependent on its geometry and the mechanical characteristics of its yielding material, and is not affected by the variation of the strength of the structure. The CYDE is based on the Yield Point Spectrum representation of seismic design demands [18,19] and a new relation between the strength and the displacement ductility \( \mu \) of a structure, proposed in this study, to quantify the inelastic displacement demand of an existing structure subjected to a selected ground motion excitation.

In contrast with the constant-period approaches, this procedure explicitly accounts for the increase of the stiffness of the structure due to an increase of its strength, thus leading to realistic values of displacement ductility demand for a wide range of structure vibration periods. The yield displacement and the yield strength of an existing structure can be determined directly and with high confidence. The vibration period of the structure emerges as an outcome of the values of the yield displacement and the yield strength of the structure. Therefore, the CYDE is a vibration period-independent inelastic displacement-based seismic evaluation procedure.

**Dynamic modelling**

The dynamics of the inelastic flexural behavior of a fixed-base structure is investigated by using a cantilever SDOF structure presented in Fig. 1. Mass \( m_s \) is the concentrated seismic mass of the structure. Its elastic stiffness and damping coefficient are defined as \( k_s, c_s \), while its post-yield stiffness is determined using the hardening coefficient \( \alpha_s \). The following quantities are defined:

1. Yield displacement \( u_{y,s} \):

\[ u_{y,s} = \frac{M_{y,s}}{3EI / H^2} = \frac{AE\varepsilon_{y,s}B}{3EI / H^2} = \frac{AE\varepsilon_{y,s}B}{3E \left( \frac{AB^2}{2} \right) / H^2} = \frac{2}{3} \varepsilon_{y,s} \frac{H^2}{B} \]  

(2)

The geometry of the structure is shown in Fig. 1, with \( A \) representing the symmetrically positioned areas of the yielding material in the cross-section of the column. These areas simulate the flanges
of a typical I-shape section or the reinforcement for a typical reinforced concrete or reinforced masonry column cross section. The yielding material is assumed to be standard structural steel.

Here, $I$ is the moment of inertia of the cross section of the SDOF system column, and $H$ is the height of the column (Fig. 1). The effect of the weight of the structure on its flexural response is neglected. It is evident that the yield displacement of the SDOF system responding in flexure depends only on the geometry of the structure (its aspect ratio and its height) and the yield strain of the yielding material.

2. Yield strength reduction factor $R^*$:

$$ R^* = \frac{F_{el,s}^*}{F_{y,s}^*} \tag{3} $$

where $F_{el,s}^*$ is the strength of the SDOF structure required for it to remain elastic under the given ground motion excitation and $F_{y,s}^*$ is one of the possible yield strengths of the inelastic SDOF structure, shown in Fig. 2. Note here the assumption that the yield displacement $u_{y,s}$ of the SDOF structure is not affected by the change of its strength.

3. Yield stiffness $k_y$:

$$ k_y = \frac{k_s}{R} \tag{4} $$
4. Elastic vibration period and cyclic frequency:

\[ T_n = 2\pi \sqrt{\frac{m_s}{k_s}} = 2\pi \sqrt{\frac{m_s}{F_{el,s}^*/u_{y,s}}}, \quad \omega_n = \sqrt{\frac{k_s}{m_s}} = \sqrt{\frac{F_{el,s}^*/u_{y,s}}{m_s}} \]  

(5)

5. Yield vibration period and cyclic frequency:

\[ T_y = 2\pi \sqrt{\frac{m_s}{k_y}} = 2\pi \sqrt{\frac{m_s}{F_{el,s}^*/(R^*u_{y,s})}}, \quad \omega_n = \sqrt{\frac{k_y}{m_s}} = \sqrt{\frac{F_{el,s}^*/(R^*u_{y,s})}{m_s}} \]  

(6)

Note that:

\[ T_y = T_n \sqrt{R^*} \]  

(7)

Figure 2. Definition of the strength reduction factor \( R^* \).

The results presented in this study were obtained by simulating the inelastic response of the fixed-base structure described above using Matlab and OpenSees models.

Comparison of the response of a stiff structure to analytical pulse excitation using different seismic design approaches

Consider a prototype SDOF steel structure with aspect ratio \( H/B=2 \), \( H=2m \), mass \( m_s=1000t \) (weight \( W_s=9810kN \), hardening coefficient \( \alpha_s=0 \) and non-hysteretic damping ratio \( \xi_s=0.001\% \). The yield displacement of this structure is \( u_{y,s}=5.3mm \) (Equation 1). The base of the structure is excited by an analytical symmetric Ricker pulse [20] with a pulse period \( T_p=0.5s \) and pulse peak acceleration \( a_p=0.25g \). The elastic vibration period corresponding to the yield displacement of the structure is \( T_n=0.22s \) (Equation 5) and the strength of the structure required for it to remain elastic under the
given excitation is $F_{el,s}^* = 3918 \text{kN}$. The objective is to evaluate the ductility demand imposed on the SDOF structure whose strength is 4 times smaller than 3918kN, i.e. $F_{y,s}^* = 979.5 \text{kN}$.

The first inelastic SDOF structure is evaluated using the constant-yield-displacement approach. The yield strength reduction factor is $R^* = 4$, and the vibration period corresponding to the yield displacement of the structure is $T_y = 0.44 \text{s}$ (Equation 6). The maximum inelastic displacement of this SDOF structure is $u_{m,s} = 46.1 \text{mm}$ and the displacement ductility demand $\mu = 8.7$.

The second inelastic SDOF system is evaluated using the conventional constant-period approach. Thus, the period of this inelastic SDOF remains $T_n = 0.22 \text{s}$. Using a strength reduction factor $R_y = 4$, the yield displacement of the structure $u_{y,s} = 1.2 \text{mm}$ (Equation 1). Under the same symmetric Ricker pulse motion the maximum inelastic displacement of this SDOF structure is $u_{m,s} = 37.9 \text{mm}$, and the resulting displacement ductility demand is $\mu = 31.57$, about four times larger than that calculated for the SDOF structure evaluated using the constant-yield-displacement approach.

![Figure 3](image_url)

Figure 3. Displacement time-history response of two SDOF systems designed using the constant-period and the constant-yield-displacement approaches.

The displacement response time histories of the two inelastic SDOF structures subjected to the symmetric Ricker ground motion excitation ($T_p = 0.5 \text{s}$ and $a_p = 0.25 \text{g}$) are shown in Fig. 3. While the maximum inelastic displacements of the two structures are not very different, the computed displacement ductility demands are significantly different. This is because the yield displacement of the structure designed using the constant-period approach is unrealistically small.

**Ground motion response data**

To further investigate the trend observed in the example, displacement ductility response spectra for a portfolio of inelastic SDOF structures designed using the constant-yield-displacement approach were computed for an ensemble of recorded ground motions.

The 80 motions used in this study cover a wide range of ground motion types, magnitudes (5.5 to 7.7) and distances (10km to 60km). They were taken from the Pacific Earthquake Engineering Research (PEER) Center next generation attenuation (NGA) strong motion database [21, 22]. The geometry of the SDOF structure, its aspect ratio $H/B$ and height $H$, and the yield
strain of the yielding material $\varepsilon_{y,s}$ are the fundamental design parameters of the SDOF structure because they determine the yield displacement $u_{y,s}$ of the structure (Equation 1).

Thus, a portfolio of SDOF structures was created by setting the height $H$, the yield strain of the yielding material $\varepsilon_{y,s}$ (thereby setting the yield displacement), the hardening coefficient $\alpha_s$ and the yield strength reduction factor $R^*$, then varying the aspect ratio $H/B$ by setting it equal to integers in the set $\{1,2,\ldots,10\}$. First, the elastic displacement spectrum for each ground motion was used to determine the corresponding elastic vibration period $T_n$ (as the shortest vibration period that corresponds to the computed SDOF structure yield displacement), followed by the calculation of $F_{el,s}^*$, the strength required for the structure to remain elastic, given its yield displacement. Second, the yield strength $F_{y,s}^*$ of each of the 10 SDOF structures in the portfolio was determined using the selected yield strength reduction factor $R^*$ to create spectra for a predetermined $R^*$ value (constant $R^*$ spectra). Third, a non-linear response time history analysis was conducted for each of the 80 ground motions and the maximum inelastic displacement of the structure and the corresponding displacement ductility values were determined.

The distributions of the displacement ductility demand values at aspect ratios $H/B$ equal to 1, 5 and 10 for the $R^*=3$, $H=2m$, $\varepsilon_{y,s}=0.2\%$ and $\alpha_s=0$ portfolio of SDOF structures are plotted in Fig. 4. The values of the means and medians of these distributions are listed in Table 1. Lognormal distributions were fit to the datasets for each aspect ratio $H/B$ and tested using the Chi-squared test.

![Figure 4. Median constant $R^*$=3 displacement ductility spectrum for the SDOF systems that yielded and the fitted lognormal distribution functions for $H=2m$, $f_{y,s}=420MPa$, $\alpha_s=0$ and $H/B$ values of 1, 5 and 10.](image)

A strength-ductility-geometry relation that approximates the computed constant $R^*$ displacement ductility demand spectra is shown in Fig. 5 and formalized in Equation 8. This $\mu$-$R^*$-$H/B$ relation gives the value of the displacement ductility demand $\mu$ corresponding to a certain strength reduction factor value $R^*$ for an inelastic SDOF structure designed using the constant-yield-displacement approach that responds in flexure to earthquake ground motion excitation.

The proposed $\mu$-$R^*$-$H/B$ relations (Equation 8) for $R^*=4$, nominal steel yield strength $f_{y,s}=235MPa$ (yield strain $\varepsilon_{y,s}=0.11\%$), and hardening ratio $\alpha_s=0$ are plotted in Fig. 5 for three different values of the SDOF structure height $H=\{1,2,4\}m$. 
\[
\mu = \begin{cases} 
\text{not applicable} & H/B \leq 1 \\
\sqrt{1 + (R^* - 1) \frac{(H/B)_c}{(H/B)}} & 1 < H/B \leq (H/B)_c \\
H/B > (H/B)_c & 
\end{cases}
\] (8)

| \(R^*\) |
|---|---|---|---|---|---|---|---|---|
| \(f_{y,s}\) (MPa) | \(\alpha_s=0\%\) | \(\alpha_s=5\%\) | \(\alpha_s=10\%\) | \(\alpha_s=0\%\) | \(\alpha_s=5\%\) | \(\alpha_s=10\%\) |
| 235 | 5m/H | 5m/H | 5m/H | 5m/H | 5m/H | 5m/H | 5m/H |
| 275 | 5m/H | 4m/H | 4m/H | 5m/H | 4m/H | 4m/H | 5m/H |
| 355 | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H |
| 420 | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H |
| 500 | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H | 4m/H |

Table 1. \((H/B)_c\) values for different values of \(R^*\), SDOF system force-deformation response hardening ratio \(\alpha_s\) and nominal steel yield strengths \(f_{y,s}\)

Figure 5. Proposed \(\mu-R^*-H/B\) relations for different heights \(H\) (m), with the strength reduction factor \(R^*=4, f_{y,s}=235\)MPa and \(\alpha_s=0\), i.e. \((H/B)_c = 5\)m/H.

**Constant yield displacement evaluation procedure**

The \(\mu-R^*-H/B\) relations makes it possible to propose a novel seismic evaluation procedure for inelastic SDOF structures that respond to ground motion excitation in flexure, the Constant Yield Displacement Evaluation (CYDE) procedure. Starting from the basic design parameters of the SDOF structure (its strength, height, aspect ratios, mass and the mechanical characteristics of its yielding material), the goal of this seismic evaluation procedure is to determine the displacement...
ductility demand $\mu$ of the existing structure subjected to earthquake ground motion excitation expected for the seismic hazard level the structure is evaluated at and compare it to the displacement ductility capacity of the existing structure to determine if the existing structure is satisfactory. The displacement ductility demand is determined as follows (Fig. 6):

1. **Determine the yield displacement** $u_{y,s}$ (Equation 1) **and the yield strength** $F_{y,s}^*$ of the SDOF model of an existing structure.
2. **Calculate** $F_{el,s}^*$, the strength required for this SDOF structure to remain elastic for the given design seismic hazard. Using the elastic displacement response spectrum for the design seismic hazard, find the shortest elastic vibration period $T_n$ corresponding to the calculated yield displacement $u_{y,s}$.
3. **Determine the** $R^*$, the yield strength reduction factor of the inelastic SDOF model of the existing structure with yield strength $F_{y,s}^*$, as $R^* = \frac{F_{el,s}^*}{F_{y,s}^*}$.
4. **Calculate the displacement ductility demand** $\mu$ of the structure from the $\mu$-$R^*$-$H/B$ relations (Equation 8).

$$\mu = \frac{u_{m,s}}{u_{y,s}}$$

**Figure 6.** Proposed CYDE procedure.

**Conclusions**

The proposed $\mu$-$R^*$-$H/B$ relations were used to develop a new displacement-based seismic evaluation procedure, the Constant Yield Displacement Evaluation (CYDE) procedure. The CYDE procedure is based on the argument that the yield displacement of the structure is constant,
that it depends on the geometry of the structure and the mechanical characteristics of the yielding material, and that it does not depend on the strength of the structure. The vibration period of the evaluated structure is obtained as an outcome of the values of the yield displacement and the yield strength of the structure, both of which can be determined with high confidence for an existing structure. The four steps of the CYDE procedure are easy to implement using the currently available seismic design spectra, elastic response analysis, and the $\mu$-$R^*$-$H/B$ relations presented in this paper. To evaluate an existing structure, the displacement ductility demand estimated using the CYDE procedure is compared to the expected displacement ductility capacity.

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