SEISMIC DESIGN OF LINEAR PASSIVE CONTROL SYSTEMS USING NONSMOOTH H_\infty SYNTHESIS

C.-M. Chang\textsuperscript{1} and C.-Y. Yang\textsuperscript{2}

ABSTRACT

Seismic design of passive control systems is sometimes critical to structural engineers, i.e., optimization of stiffness and damping coefficient in these systems. Thus, when performing any optimization algorithms to design passive control systems, some constraints should be included in the design parameters. Therefore, this study proposes a new seismic design method for determining the design parameters of passive control systems used in a multiple degree-of-freedom structure. In this method, an active control algorithm, namely the nonsmooth H_\infty synthesis algorithm, is employed to design the passive control parameters, such as multiple linearly viscous dampers in a building, a seismic isolation system at the base of a building, or multiple tuned mass dampers in a building. A feasible range of stiffness and damping coefficients are first given in this algorithm. The seismic characteristics are represented by the Kanai-Tajimi spectrum. Through this active control algorithm, the optimal parameters can be determined by minimizing the H_\infty norm of responses in the objective function, which can be a combination of structural responses. Finally, effective passive control systems are designed for a specific structure. Various numerical examples are provided to exhibit control performance of passively controlled structures designed by the proposed method and are compared to the design examples illustrated in the literature. The examples considered in this study are multiple linearly viscous dampers installed in a building when the installing locations are known, a base isolation system for a building as the mass at the isolation layer is given, and multiple tuned mass dampers in a building with known locations and masses. As a result, the control effectiveness of the passive control systems designed by the proposed method is comparable or even superior to those systems designed by the method in previous studies.

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Seismic Design of Linear Passive Control Systems Using Nonsmooth H_∞ Synthesis

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ABSTRACT

Seismic design of passive control systems is sometimes critical to structural engineers, i.e., optimization of stiffness and damping coefficient in these systems. Thus, when performing any optimization algorithms to design passive control systems, some constraints should be included in the design parameters. Therefore, this study proposes a new seismic design method for determining the design parameters of passive control systems used in a multiple degree-of-freedom structure. In this method, an active control algorithm, namely the nonsmooth H_∞ synthesis algorithm, is employed to design the passive control parameters, such as multiple linearly viscous dampers in a building, a seismic isolation system at the base of a building, or multiple tuned mass dampers in a building. A feasible range of stiffness and damping coefficients are first given in this algorithm. The seismic characteristics are represented by the Kanai-Tajimi spectrum. Through this active control algorithm, the optimal parameters can be determined by minimizing the H_∞ norm of responses in the objective function, which can be a combination of structural responses. Finally, effective passive control systems are designed for a specific structure. Various numerical examples are provided to exhibit control performance of passively controlled structures designed by the proposed method and are compared to the design examples illustrated in the literature. The examples considered in this study are multiple linearly viscous dampers installed in a building when the installing locations are known, a base isolation system for a building as the mass at the isolation layer is given, and multiple tuned mass dampers in a building with known locations and masses. As a result, the control effectiveness of the passive control systems designed by the proposed method is comparable or even superior to those systems designed by the method in previous studies.

Introduction

Passive control has been widely accepted as an effective means of protecting structures against earthquakes. The commonly seen passive control strategies include installing viscous dampers between floors in a building, equipping isolation bearings at the base of a building, and placing several harmonic absorbers in a building [1]. Each passive control strategy contains stiffness and/or damping coefficients to be determined for the control devices. High performance can be achieved if the control devices used are optimally designed.

Passive base isolation control is a technique used to employ a flexible device underneath the structure which shifts the dominant frequency of the structure away from the frequencies

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having significant magnitudes in excitations [2-3]. Moreover, as indicated in the Uniform Building Code [4], supplemental damping devices to accommodate excessive base displacements under a stronger maximum credible earthquake (MCE) excitation are of need [5-6]. In design, both stiffness and damping terms should be concurrently determined in order to achieve optimal control effectiveness for a seismically base-isolated building.

Buildings with viscous dampers allow input energy from earthquakes to be dissipated, resulting in reduced structural responses. Thus, a number of researchers have developed methods to optimally design viscous dampers in a building. For example, Constantinou and Tadjbakhsh [7] designed an optimal viscous damper installed on the first floor in a shear-type building. Garcia [8] simplified sequential search algorithm to directly determine damper configurations in multiple-degree-of-freedom (MDOF) structures. Fu and Kazuhiko [9] exploited the assumed mode method to determine the optimal damping needed in a building. Moreover, some researchers employed the gradient-based optimization approach to determine the optimal damping within a limited range [10-13]. Therefore, constraints added to the optimization process yield more realistic viscous dampers to be designed in structures.

Buildings with multiple tuned mass dampers (MTMD) attached offered response reductions over a broader bandwidth, allowing more structural modes to be tuned. However, designing multiple tuned mass dampers in a building is still a challenging problem. For instance, Li [14] and Li and Liu [15] proposed a series of multiple tuned mass damper models and designed them in accordance with the physical or modal parameters with some constraints. Some researchers employed optimization methods such as the gradient-based algorithms to determine a number of design variables for a set of MTMD [16-18]. Moreover, Lin et al. [19] considered a more realistic approach for MTMD design with limited strokes in a building and carried out an experimental validation. When accounting for multiple tuned mass dampers, added constraints to design parameters can expedite the optimization process and yield convergent solutions.

Some active control algorithms can be applied to design a passive control system due to the stabilization nature and high performance. In the $H_2$-norm sense, Kosut [20] employed the linear quadratic regulator algorithm to derive a static output feedback controller. This algorithm allows direct design of stiffness and damping terms in a passive control system such as the isolation system or multiple viscous dampers in a structure. Zuo and Nayfeh [21] derived a static output feedback controller based on the $H_\infty$ control algorithm, which yields concurrent design of stiffness and damping terms in a passive control system. Chang et al. [23] proposed a two-stage method that accounted for both $H_2$- and $H_\infty$-norm responses to derive a static output feedback controller. The proposed method was applied to design a base isolation system for a building. All these methods design an active controller based on the stochastic responses of structures, while these methods are also applicable for designing a passive control system by adding some constraints. Moreover, these design methods can be further extended to derive a dynamic output feedback controller, which can be employed for the design of MTMD.

In this study, a design method of passive control systems is developed based on the nonsmooth $H_\infty$ synthesis. This nonsmooth $H_\infty$ synthesis method was proposed by Apkarian and
Noll [24], while this method can be easily implemented in MATLAB [25-26]. The method performs the numerically optimization process in the $H_\infty$-norm sense, and the added constraints are considered for all design variables. This optimization process yields either a static output feedback controller or a dynamic output feedback controller, which fits the need of passive control design for structures. In addition to this optimization method, the other objective function is included to account for the scale issue in the $H_\infty$-norm responses (i.e., structural displacements and absolute accelerations have a different order of magnitude), and this objective function can be formed by $H_2$- or $H_\infty$-norm responses with additional scaling variables. Using this two-stage approach, an optimally designed passive control system can be determined. Three examples are studied in this research including the design of a base isolation system for a building, of various linearly viscous dampers in a building, and of multiple tuned mass dampers in a building. All the results obtained from the proposed method are compared to the optimized solutions from previous studies. As seen in the comparative results, the proposed method is effective for designing an optimal passive control for structures.

Problem Formulation

Three types of passive control systems are presented including the base isolation system, multiple viscous damper system, and multiple tuned mass damper system in buildings. To integrate the nonsmooth $H_\infty$ synthesis algorithm into the design, the passive control force should be formulated as a function of structural outputs. Meanwhile, this force can be computed from a static form (e.g., viscous damper forces) or a dynamic form (e.g., multiple tuned mass dampers). For a base-isolated building, the shear force can be a combination of spring and damping forces from the isolation layer to the structure. Thus, the isolation system is subsequently implemented as a static output feedback controller. In this study, the isolation system is modeled by this approach. Note that the superstructure with a zeroed isolation shear force contains a rigid-body mode. Each passive control design is individually introduced in the followings.

Base Isolated Building

An $n$-DOF shear-type building with a base isolation system can be modeled by

$$\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{b}\dot{u} - \ddot{\mathbf{M}}\ddot{x}_b$$  \hspace{1cm} (1)

where

$$\mathbf{b} = [-1 \ 0 \ \cdots \ 0]^T, \ \mathbf{1} = [1 \ 1 \ \cdots \ 1]^T, \ \mathbf{x} = [x_b \ x_1 \ \cdots \ x_n]^T$$

$\ddot{\mathbf{M}}, \ \ddot{\mathbf{C}}, \ \ddot{\mathbf{K}}$ are the mass, damping, and stiffness matrix of the superstructure including the coupling terms with the seismic isolation; the subscript “$b$” denotes the base layer and the subscript “$i$”, $i \in [1,n]$, denotes the $i$-th floor; $\ddot{x}_b$ is the absolute ground acceleration; $\dot{u}$ is the control force input provided by seismic isolation and is equal to $k_b\dot{x}_b + c_b\dot{\ddot{x}}_b$, where the stiffness $k_b$ and damping coefficient $c_b$ are treated as unknowns to be designed in this study [23]. This equation can be further converted into a state-space equation and given by
\[
\dot{z} = A_{iso} z + B_{iso} u + E_{iso} \ddot{v}_g
\]
\[
u = C_{1,iso} z
\]
\[
y = C_{2,iso} z + D_{2,iso} u + F_{2,iso} \ddot{v}_g
\]

where
\[
z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad A_{iso} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_{iso} = \begin{bmatrix} 0 \\ M^{-1}b \end{bmatrix}, \quad E_{iso} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad C_{1,iso} = \begin{bmatrix} k_b & 0 & \hdots & 0 & c_b & 0 & \hdots & 0 \end{bmatrix}
\]

\(C_{2,iso}, D_{2,iso},\) and \(F_{2,iso}\) are related to the outputs and can be formed accordingly.

**Building with Multiple Viscous Dampers**

The model of an \(n\)-DOF building with \(m\) viscous dampers can be written by

\[
M\ddot{x} + Cx + Kx = bu - M\ddot{v}_g
\]

where
\[
1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T, \quad x = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}^T
\]

\(M, C, \) and \(K\) are the mass, damping, and stiffness matrix of the building; \(b\) is relevant to the number and locations of viscous dampers; \(u\) is the damper forces. Similarly, the equation of motion in Eq. (3) can be presented by a state-space equation as

\[
z = A_{vis} z + B_{vis} u + E_{vis} \ddot{v}_g
\]
\[
u = C_{1,vis} z
\]
\[
y = C_{2,vis} z + D_{2,vis} u + F_{2,vis} \ddot{v}_g
\]

where
\[
z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad A_{vis} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_{vis} = \begin{bmatrix} 0 \\ M^{-1}b \end{bmatrix}, \quad E_{vis} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}
\]

The rows in \(C_{1,vis}\) are also related to the locations of the viscous dampers, and the nonzero terms are the viscous damping coefficients at the entries related to velocities. Meanwhile, \(C_{2,vis}, D_{2,vis},\) and \(F_{2,vis}\) are related to the outputs and can be formed accordingly. Both Eqs. (2) and (4) has a similar form because the control forces are formed by static output feedback.

**Building with Multiple Tuned Mass Dampers**

An \(n\)-DOF building with \(m\) tuned mass dampers can be expressed by

\[
M\ddot{x} + Cx + Kx = bu - M\ddot{v}_g
\]
\[
M_{tmd} \ddot{x}_{tmd} + C_{tmd} \dot{x}_{tmd} + K_{tmd} x_{tmd} = -b_{tmd} \left( \ddot{x} + 1i_g \right)
\]

where
\[
1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T, \quad x = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}^T, \quad x_{tmd} = \begin{bmatrix} x_{1,tmd} & \cdots & x_{m,tmd} \end{bmatrix}^T, \quad u_j = k_{i,tmd} x_{i,tmd} + c_{i,tmd} \dot{x}_{i,tmd}
\]
\( \textbf{M} \), \( \textbf{C} \), and \( \textbf{K} \) are the mass, damping, and stiffness matrix of the building; \( \textbf{b} \) is relevant to the number and locations of TMDs; \( \textbf{u} \) is the TMD forces; \( \textbf{M}_{tmd} \), \( \textbf{C}_{tmd} \), and \( \textbf{K}_{tmd} \) are the diagonal mass, damping, and stiffness matrix of TMDs; \( u_{i,tmd} \) is the \( i \)-th TMD force; \( x_{i,tmd} \) is the \( i \)-th TMD displacement relative to the connecting floor; \( k_{i,tmd} \) and \( c_{i,tmd} \) are the stiffness and damping coefficient of the \( i \)-th TMD. When converting in Eq. (5) into a state-space representations, both building and TMDs are individually formulated and given by

\[
\dot{\textbf{z}} = \textbf{A} \textbf{z} + \textbf{B} \textbf{u} + \textbf{E} \ddot{\textbf{y}}_g
\]

and

\[
\dot{\textbf{y}}_{tmd} = \textbf{A}_{tmd} \textbf{z}_{tmd} + \textbf{B}_{tmd} \textbf{y}_{tmd}
\]

where

\[
\textbf{z}_s = \begin{bmatrix} \dot{\textbf{x}} \\ \ddot{\textbf{x}} \end{bmatrix}, \quad \textbf{A}_s = \begin{bmatrix} 0 & \textbf{I} \\ -\textbf{M}^{-1} \textbf{K} & -\textbf{M}^{-1} \textbf{C} \end{bmatrix}, \quad \textbf{B}_s = \begin{bmatrix} 0 \\ \textbf{M}^{-1} \textbf{b} \end{bmatrix}, \quad \textbf{E}_s = \begin{bmatrix} 0 \\ \textbf{-I} \end{bmatrix}, \quad \textbf{y}_{tmd} = \ddot{\textbf{x}} + \textbf{I} \ddot{\textbf{y}}_g
\]

\[
\textbf{z}_{tmd} = \begin{bmatrix} \textbf{x}_{tmd} \\ \textbf{y}_{tmd} \end{bmatrix}, \quad \textbf{A}_{tmd} = \begin{bmatrix} 0 & \textbf{I} \\ -\textbf{M}_{tmd}^{-1} \textbf{K}_{tmd} & -\textbf{M}_{tmd}^{-1} \textbf{C}_{tmd} \end{bmatrix}, \quad \textbf{B}_{tmd} = \begin{bmatrix} 0 \\ -\textbf{M}_{tmd}^{-1} \textbf{b}_{tmd} \end{bmatrix}
\]

\( \textbf{C}_{1,s} \) and \( \textbf{D}_{1,s} \) are the matrices that allow computing the absolute floor accelerations; \( \textbf{C}_{2,s}, \textbf{D}_{2,s}, \) and \( \textbf{F}_{2,s} \) are the corresponding matrices for output measurements; \( \textbf{C}_{1,tmd} \) is a matrix presenting the TMD locations and consisting of nonzero terms with TMD stiffness and damping coefficients. In this case, Eq. (7) is the feedback controller for the primary system in Eq. (6), which is a dynamic output feedback controller.

**Closed-Loop System Representation**

Eqs. (2) and (4) are a structural system with static output feedback, while Eqs. (6-7) present a structural system with dynamic output feedback. These structure-control systems can be also presented by a closed-loop system using the transfer function forms; in the meantime, the seismic effect can be included by means of the Kanai-Tajimi spectrum. Thus, the overall systems for design are written by

\[
\textbf{Y}(s) = \begin{bmatrix} \textbf{C}_{2,iso} + \textbf{D}_{2,iso} \textbf{C}_{1,iso} \\ \textbf{H}_{iso}(s) + \textbf{F}_{2,iso} \textbf{H}_{iso}(s) \end{bmatrix} \textbf{W}(s)
\]

\[
\textbf{H}_{iso}(s) = (s\textbf{I} - \textbf{A}_{iso} - \textbf{B}_{iso} \textbf{C}_{1,iso} )^{-1} \textbf{E}_{iso} \textbf{H}_{iso}(s)
\]

\[
\textbf{Y}(s) = \begin{bmatrix} \textbf{C}_{2,vis} + \textbf{D}_{2,vis} \textbf{C}_{1,vis} \\ \textbf{H}_{vis}(s) + \textbf{F}_{2,vis} \textbf{H}_{vis}(s) \end{bmatrix} \textbf{W}(s)
\]

\[
\textbf{H}_{vis}(s) = (s\textbf{I} - \textbf{A}_{vis} - \textbf{B}_{vis} \textbf{C}_{1,vis} )^{-1} \textbf{E}_{vis} \textbf{H}_{vis}(s)
\]

\[
\textbf{Y}(s) = \begin{bmatrix} \textbf{I} - \textbf{D}_{2,s} \textbf{C}_{1,tmd} \textbf{H}_{iso}(s) \\ \textbf{T}_{iso} \end{bmatrix}^{-1} \textbf{C}_{2,s} \textbf{H}_{iso}(s) + \textbf{F}_{2,s} \textbf{H}_{iso}(s) \textbf{W}(s)
\]

\[
\textbf{H}_{iso}(s) = \begin{bmatrix} \textbf{I} - \textbf{D}_{2,s} \textbf{C}_{1,tmd} \textbf{H}_{iso}(s) \\ \textbf{T}_{iso} \end{bmatrix}^{-1} \textbf{C}_{2,s} \textbf{H}_{iso}(s) + \textbf{F}_{2,s} \textbf{H}_{iso}(s)
\]
\( H_{y_y}(s) = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \)  \hspace{1cm} (11)

where \( s \) is the Laplace variable, and \( T_{tm} \) is the transformation matrix from \( y \) to \( y_{tm} \) in Eq. (7). Eq. (8) is converted from Eq. (2) for the base-isolated building; Eq. (9) is converted from Eq. (4) for the building with multiple viscous dampers; Eq. (10) is converted from Eqs. (6-7) for the building with multiple tuned mass dampers; \( H_{y_y}(s) \) is the Kanai-Tajimi spectrum of which \( \zeta \) and \( \omega_n \) are the spectrum parameters (see [23]). The objective of study is to derive minimum \( H_\infty \)-norm responses of \( Y(s) \) in Eqs. (8-10) when the optimal design parameters are obtained, i.e., the stiffness and damping coefficient in a base isolation system, the damping coefficients of viscous dampers, and the TMD stiffness and damping coefficients.

**Nonsmooth \( H_\infty \) Synthesis**

This study applies the nonsmooth \( H_\infty \) synthesis method [24] to obtain the minimum \( H_\infty \)-norm responses while determining the parameters in a passive control system. This method can be simply carried out in MATLAB [25-26] that numerically computes the minimum \( H_\infty \) norms by

\[ \min_{x \in \mathbb{R}^k} \| Y(j\omega, x_i) \|_\infty, \ \omega \in [0, \infty] \]  \hspace{1cm} (12)

where \( j = \sqrt{-1} \); \( \omega \) is the frequency; \( x_i \) is the \( k \) design variables to be determined when the minimum of Eq. (12) is achieved. The frequency response \( Y \) is identical to those in Eq. (8-10). Moreover, because \( Y \) is an output vector in the frequency domain, the \( H_\infty \)-norm response of this vector is written by

\[ \| Y(j\omega, x_i) \|_\infty = \sum_{i=1}^{m} \| Y_i(j\omega, x_i) \|_\infty, \ Y(j\omega, x_i) = \left[ Y_1(j\omega, x_i) \ldots Y_z(j\omega, x_i) \right]^T \]  \hspace{1cm} (13)

In Eq. (13), the \( H_\infty \)-norm response is unfair if the output \( y \) with \( my \) components in Eqs. (2), (4), and (6) contains different types of measurements. Thus, a modified version of Eq. (12) is employed and given by

\[ \min_{x \in \mathbb{R}^k} \sum_{i=1}^{m} \alpha_i \| T_i Y(j\omega, x_i) \|_\infty, \ \omega \in [0, \infty] \]  \hspace{1cm} (14)

where Eq. (14) presents a weighted sum of \( mo \) \( H_\infty \)-norm responses; \( T_i \) is a user defined transformation matrix; \( \alpha_i \) is the weight with respect to each \( H_\infty \)-norm response. Subsequently, \( x_c \) becomes a function of \( \alpha \in \mathbb{R}^{m_0} \), and an additional criterion is required to determine the optimal \( x_c \) such as

\[ \min_{x \in \mathbb{R}^k} \sum_{i=1}^{m_0} \left[ T Y(j\omega, x_c(u)) \right]_\infty, \ \omega \in [0, \infty] \]  \hspace{1cm} (15)

or

\[ \min_{x \in \mathbb{R}^k} \sum_{i=1}^{m_0} \left[ T Y(j\omega, x_c(u)) \right]_\infty, \ \omega \in [0, \infty] \]  \hspace{1cm} (16)
where $m_{\alpha}$ is the number of $\alpha$ used to find the minimum $H_2$ or $H_\infty$ norm. Finally, utilizing Eq. (15) or (16) can render the optimal design of a passive control system. In this study, Eq. (16) is employed to determine the design variables in passive control systems.

**Numerical Examples**

This study presents three numerical examples regarding the design of base isolation systems, multiple viscous damper systems, and multiple tuned mass damper systems. All design results are compared to the previous studies. For example, the example of base-isolated buildings refers the experimental building model in [2], in which a low-damping isolation system was used. The Kanai-Tajimi spectrum used in Eq. (8) is identical to the one in [23]. The inputs $T_Y$ used in Eqs. (14-15) are the base displacement and roof acceleration. The Kanai-Tajimi spectrum used is determined by the scaled 1999 Chi-Chi earthquake records at the stations of CHY014, CHY028, CHY088, and TCU071. The time-domain responses are compared with the design in [2] as shown in Figure 1. In this figure, Type I-IV isolation systems are the optimal design from the proposed method, the optimal isolation system in [23], the isolation system in [2], and the isolation system in [2] with an effective damping ratio of 20%. The proposed method renders better reduction on roof accelerations among all isolation systems under most earthquake excitations. Therefore, the proposed method is applicable to design a base isolation system for a building.

![Figure 1](image_url)

**Figure 1.** Time-domain responses of various isolation systems under (a) scaled and (b) original earthquake excitation.

Another example is the proposed method applied for the design of multiple viscous dampers in a building. The building selected is the 20-story building in the ASCE benchmark control problem [27]. In this example, the number of linearly viscous dampers used is 4, 2, and 2 in the 1st-3rd floors, respectively, and all these dampers have the same damping coefficient. Only one linearly viscous damper per floor is employed in the 4th-20th floors, and these dampers have...
another number of damping coefficient. After performing the proposed optimization process, the time-domain performance is shown in Figure 2. The proposed method (i.e., Type I damper system in the figure) is also compared to the previous study (i.e., Type II damper system in the figure) in [28], of which the damper configuration is the same and the damping coefficient is linearized to 2,750 kN-s/m. The responses used in Eq. (16) is the roof displacement and acceleration. As can be seen in Figure 2, the proposed method yield better performance than the viscous damper system in [28] in terms of roof displacement and acceleration. For linearly viscous damper systems, the proposed method still provides effective results to be attained.

Figure 2. Performance evaluation of two viscous damper systems under (a) scaled and (b) original earthquake excitation.

The last example to be presented is the optimal design of multiple tuned mass dampers in a building. Figure 3 compares control performance of the optimally designed TMD system from the proposed method to the previous study in [19]. In this case, the responses used in Eq. (16) is the roof displacement and acceleration. In this figure, Type I-III TMD systems are the optimal design from this study, the TMD system with two resonant frequencies in [19], and the TMD system with only one resonant frequency in [19]. As shown in this figure, the optimal TMD system derived from the proposed method outperforms those systems in the previous study in most seismic events, in particular of the root-mean-square responses. Therefore, the proposed method can determine the optimal passive control system via not only static output feedback but also dynamic output feedback.

Conclusions

This study presented a new method to design a passive control system for a building using the nonsmooth \( H_\infty \) synthesis algorithm proposed in [25]. To implement this new method, the passive control system should be modeled as either a static output feedback controller or a dynamic output feedback controller. Then, a two-stage optimization process was conducted to first find the a set of solutions with respect to the weight in Eq. (14). The second stage was to determine
the optimal design of passive control systems by Eq. (15) in the $H_\infty$-norm sense. Three examples were studied to numerically validate the proposed method. As seen in the results, the proposed method allowed high-performance passive control systems to be obtained. This method is applicable to design several variables in a control system such as the examples of multiple viscous dampers and multiple tuned mass damper in a MDOF building.

![Figure 3](image)

**Figure 3.** Performance evaluation of various TMD systems under (a) scaled and (b) original earthquake excitation.

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