NONLINEAR FE ANALYSIS OF CONCRETE DIAPHRAGMS SUBJECTED TO BACKSTAY FORCES IN HIGH-RISE CORE WALL BUILDINGS

M. Mahmoodi$^1$ and P. Adebar$^2$

ABSTRACT

An important part of the seismic design of high-rise core wall buildings involves estimating the forces that develop within the diaphragms that connect the tower walls to a podium structure or below-grade basement walls. These diaphragms are usually modelled as elastic members, and the choice of effective stiffness significantly influences how much force will go into the backstay force path (podium or below-grade walls) versus the tower wall foundations. Given the difficulty in making accurate estimates, upper and lower-bound stiffness properties are sometimes used to bound the solution. In the current study, nonlinear finite element (NLFE) model VecTor2, which utilizes a state-of-the-art material model for cracked-concrete subjected to in-plane shear and normal forces, was used to study the reduction in shear and flexural stiffnesses of concrete diaphragms subjected to backstay forces. The analytical model was verified by comparing predictions against the results of a model diaphragm test. NLFE analysis was used to examine the influence of amount of diaphragm reinforcement (ranging from 0.5% to 2.0%) and relative spans of the diaphragm (parallel and perpendicular to the backstay forces). The stiffness of the diaphragm remains constant until significant diagonal cracking occurs, which typically only reduces the stiffness of the diaphragm by about 20%. Much larger reductions in both shear and flexural stiffnesses occur when strong-axis bending of the diaphragm results in a large zone of tension cracks in the “flexural tension zone.” An important observation is that the shear and flexural stiffnesses of the diaphragms degrade simultaneously. Depending on the relative diaphragm spans, the shear deformations were found to contribute between 70% and 90% of the total diaphragm deflection.

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An important part of the seismic design of high-rise core wall buildings involves estimating the forces that develop within the diaphragms that connect the tower walls to a podium structure or below-grade basement walls. These diaphragms are usually modelled as elastic members, and the choice of effective stiffness significantly influences how much force will go into the backstay force path (podium or below-grade walls) versus the tower wall foundations. Given the difficulty in making accurate estimates, upper and lower-bound stiffness properties are sometimes used to bound the solution. In the current study, nonlinear finite element (NLFE) model VecTor2, which utilizes a state-of-the-art material model for cracked-concrete subjected to in-plane shear and normal forces, was used to study the reduction in shear and flexural stiffnesses of concrete diaphragms subjected to backstay forces. The analytical model was verified by comparing predictions against the results of a model diaphragm test. NLFE analysis was used to examine the influence of amount of diaphragm reinforcement (ranging from 0.5\% to 2.0\%) and relative spans of the diaphragm (parallel and perpendicular to the backstay forces). The stiffness of the diaphragm remains constant until significant diagonal cracking occurs, which typically only reduces the stiffness of the diaphragm by about 20\%. Much larger reductions in both shear and flexural stiffnesses occur when strong-axis bending of the diaphragm results in a large zone of tension cracks in the “flexural tension zone.” An important observation is that the shear and flexural stiffnesses of the diaphragms degrade simultaneously. Depending on the relative diaphragm spans, the shear deformations were found to contribute between 70\% and 90\% of the total diaphragm deflection.

\textbf{Introduction}

Concrete shear walls are a popular lateral force resisting systems for high-rise buildings as they provide good lateral drift control when the building is subjected to earthquake motions or wind forces, and are relatively simple to construct. In North America, the shear walls are commonly located around a central cluster of elevator and stairway shafts to form the core of the building. Core walls typically extend from the top of the tower all the way down to the foundation level. In the upper levels, the concrete floor slabs can be modelled as rigid diaphragms that force the shear walls and gravity-load columns to undergo the same lateral deformation. In the lower levels of the buildings, concrete floor slabs interconnect the core (tower) walls with other structural components that have the potential to resist lateral force and overturning moment.

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If the lower-level floor slabs within a podium structure or below grade are modelled as rigid diaphragms, a large portion of the lateral force and the overturning moment in the core walls will be transferred into the other portions of the structure such as the perimeter foundation walls, rather than be transferred down to the foundation supporting the core walls. This is commonly referred to as the “backstay effect.” If the flexibility of the diaphragms are accounted for, particularly when some cracking of the diaphragms occurs, the lateral force and overturning moment that is transferred out of the core walls will be significantly reduced and the demands on the foundation of the core walls increased [1, 2].

In order to evaluate the backstay effect in buildings, ATC-72-1 [3] suggests the use of upper bound and lower-bound stiffnesses when concrete diaphragms are modelled as linear elastic elements with a reduced effective stiffness to account for cracking. For in-plane shear stiffness of diaphragms, ATC-72-1 [3] recommends using $0.5G_cA_g$ for the upper bound, and between $0.05G_cA_g$ and $0.2G_cA_g$ for the lower-bound shear stiffness. For strong-axis flexural stiffness, ATC-72-1 [3] recommends $0.5E_cI_g$ and $0.2E_cI_g$ for the upper and lower bounds, respectively.

LATBSDC [4] and PEER [5] provide a performance-based approach for seismic design of tall buildings using three-dimensional nonlinear dynamic response analysis for two hazard levels. The in-plane shear and flexural stiffnesses recommended for non-pretensioned diaphragms are $0.4E_cA_g$ ($1.0G_cA_g$) and $0.5E_cI_g$ for service-level evaluation and $0.1E_cA_g$ ($0.25G_cA_g$) and $0.25E_cI_g$ for MCE-level evaluation, respectively. LATBSDC [4] also suggests conducting two sets of analyses to evaluate backstay effects using upper bound and lower-bound stiffness assumptions for floor diaphragms at the podium and below. They recommend the use of $0.5G_cA_g$ for the upper bound and $0.25G_cA_g$ for the lower-bound in-plane shear stiffness. For the in-plane flexural stiffness of diaphragms, they recommend using $0.25E_cI_g$ for the upper bound and $0.1E_cI_g$ for the lower-bound flexural stiffness.

The choice of effective stiffness used for modelling the diaphragm has a very significant influence on the backstay forces. Very limited test results are available for the behavior of concrete diaphragms subjected to large in-plane loads such as backstay forces, therefore nonlinear finite element analysis was used in the current study to investigate the behavior of concrete diaphragms subjected to backstay forces.

**Nonlinear Finite Element Analysis**

Nonlinear finite element analysis was used to investigate the reduction in shear and flexural stiffness of concrete diaphragms subjected to in-plane forces. Computer program VecTor2 [7] was used for the analysis as it utilizes state-of-the-art material models for cracked reinforced concrete elements subjected to in-plane shear and normal forces. VecTor2 [7] is based on the disturbed stress field model [8], which is a refinement of the modified compression field theory [9]. In VecTor2 [7], the concrete model accounts for the reduction in concrete compression strength and stiffness due to tensile straining and transverse cracking. The model also accounts for the crack shear-slip deformations by relating shear slip to local shear stresses along cracks. The model relates the post-cracking rotation of the principal stress field to the post-cracking rotation of the principal field using...
a rotation lag. The post-cracking tensile stresses between cracks due to bond between concrete and reinforcement (tension stiffening) are included in the model. In addition, the model accounts for the reduction in concrete cracking strength due to transverse compressive stresses. Nonlinear functions are used to describe the stress-strain relationship of concrete in compression and tension. The constitutive model used for reinforcement is trilinear, consisting of a linear-elastic response, a yield plateau, and a linear strain-hardening phase until rupture. The 4-node plane stress rectangular element of VecTor2 was used in the current study to model concrete with distributed reinforcement in both horizontal and vertical directions representing various parts of the structure.

Validation of FE Model

In order to validate the NLFE model, analysis results were compared with the results of a 1:4.5 scale concrete diaphragm test by Nakashiam [6]. The test specimen represented an interior panel of a floor system in a building. It was supported by a shear wall on one edge and by columns on the opposite edge. Overhanging slabs, equal to one quarter of the panel dimension, were added on three sides to represent parts of the floor slabs of the adjacent bays. The basic panel was 1630 × 1630 mm and 40 mm thick. Details of the arrangement and properties of reinforcing bars are given in Nakashiam [6]. The compressive strength of concrete was 27.6 MPa for the floor slabs and the walls and 34.5 MPa for the columns. For the test of interest, the support condition provided to the wall was to prevent the wall from moving in the floor plane, while the columns were supported in free-to-slide conditions. The in-plane load was applied along the column line, parallel to the shear wall. To simulate the desired shear action, the in-plane load was equally distributed among five uniformly spaced studs embedded along the loading line. The in-plane deflection of the slab along the loading line was measured.

In the VecTor2 analytical model, fourteen concrete material types with different amounts of reinforcement in the two horizontal directions were used to represent various regions of the slab in the finite element model. Thirty-eight regions were created to represent the complicated arrangement of rebars in the slab and beams. The beams and the wall were modelled using elements with thicknesses equal to the beam depth and the wall height, respectively. A finite element mesh with a mesh size of 34 × 34 mm was used for both slab and beams. The total mesh consisted of approximately 4680 nodes and 4530 rectangular elements.

Since the scale ratio of 1:4.5 was used for the test specimen, the self-weight of the slab specimen was small as compared to the prototype floor slab and did not cause any cracks in the slab. Thus, the influence of the slab self-weight was neglected in the analysis. A monotonic in-plane load in displacement-controlled mode with increment of 0.05 mm was applied to five nodes evenly spaced along the center line of the beam parallel to the shear wall (the loading line). The in-plane load was applied in very small increments since the model experienced a high degree of non-linearity under the loading conditions. The horizontal displacement of the slab was restrained along the wall length while the vertical displacement was fixed at one node at the bottom edge of the wall.

A few pre-existing cracks were reported in the tested slab. These cracks could have been due to concrete shrinkage and/or the effect of other preliminary tests performed on the slab specimen. The influence of pre-existing cracks was taken into account in the VecTor2 model by reducing the tensile strength of concrete to one-third of the nominal cracking strength. Fig. 1
compares the measured load-deformation response with the analytical prediction using the reduced tensile strength. There is good agreement between the experimental and analytical results. The effect of shrinkage cracking was not included in the parametric study; thus the reduction in tensile strength of concrete was not applied in the other analyses.

In addition to predicting the load-deformation response, the VecTor2 model did an excellent job of predicting the crack pattern observed in the test, which included 45-degree shear cracks between the loading line and the slab-wall junction. Flexural cracks also developed at the slab edge and extended in the slab parallel to the wall where some horizontal reinforcement was cut off. The VecTor2 model also correctly predicted that flexural failure controlled the capacity of the diaphragm.

Figure 1. Comparison of the analytical and experimental load-deformation relationships

Analytical Study

The VecTor2 NLFE model was used to investigate how a number of important parameters influence the effective stiffness of diaphragms subjected to large backstay forces. One of the main parameters was the amount of uniformly distributed reinforcement in the two span directions of the diaphragms. Three different reinforcement ratios ($\rho = A_s/A_g$) of 0.5%, 1.0% and 2.0%, representing lightly reinforced to heavily reinforced diaphragms, were investigated.

The span length of the diaphragms was another important parameter that was investigated. In the current study, the size of the core walls was assumed constant. The length of the core walls in the direction of the applied backstay forces $L_w$ was assumed to be 7 m (see Fig. 2a). The length of the diaphragm parallel to the applied backstay forces and parallel to $L_w$ is called $L_{SD}$. If the diaphragm is modelled as a beam, this span direction would be analogous to the shear depth ($SD$) of the beam model. The clear span of the diaphragm from the core to the foundation walls resisting the backstay forces is called $L_{SS}$ (see Fig. 2a). In the beam model of the diaphragm, this length is equivalent to the shear span ($SS$). Finally, the overall span length of the diaphragm in this direction is called $L$. Table 2 summarizes the four different combinations of diaphragm spans that
were investigated. Each one of these cases was investigated with the three different reinforcement amounts resulting in 12 different diaphragms that were investigated in the current study.

Further details of the diaphragms are summarized in the following. The thickness of all diaphragms \((t_d)\) was 200 mm. The foundation walls, surrounding diaphragms had a uniform thickness \((t_w = 300 \text{ mm})\). The thickness of the core wall was 400 and 600 mm in vertical and horizontal directions, respectively. The center-to-center height between floor diaphragms was assumed to be 3 m. The concrete diaphragms were reinforced with two layers of uniformly distributed reinforcement, 10M@200 mm, 15M@200 mm and 15M@100 mm in both vertical and horizontal directions, which resulted in the 0.5%, 1% and 2% reinforcement amount, respectively. The same reinforcement amount was provided to the foundation walls in longitudinal and transverse directions, while the core wall was provided with 3% longitudinal and transverse reinforcement to avoid extensive cracking and failure in the core wall.

![Figure 2](image_url)

Figure 2. Concrete diaphragms investigated in current study, including core walls and perimeter foundation walls below grade: (a) plan view of overall diaphragm; (b) plan view of half-diaphragm model used due to symmetry; (c) elevation view of one diaphragm (section A-A)

<table>
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<tr>
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<td>1.4</td>
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<tr>
<td>(L_{SS}/L_{SD})</td>
<td>0.9</td>
<td>0.6</td>
<td>0.6</td>
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The boundary conditions used in the two-dimensional model of the diaphragm are illustrated in Fig. 2(b). As the overall diaphragm is symmetrical, restraints in the horizontal direction (perpendicular to backstay forces) were introduced at the edges of the analyzed diaphragm at mid-span. The mechanism of transferring backstay forces from the foundation walls to the foundation is not completely known and requires performing three-dimensional nonlinear analysis. To eliminate the complexity of this mechanism and to simplify the analysis to a two-dimensional model, a uniformly distributed shear force was applied at the foundation wall along the diaphragm shear depth ($L_{SD}$), as shown in Fig. 2(b). This force simulated the reaction of the support. According to NIST [10], it is reasonable to assume that the diaphragm shear is distributed uniformly along the width of the diaphragm when the diaphragm has chord reinforcement located near the extreme flexural tension edge of the diaphragm. In addition, the displacement at the bottom of the core wall was restrained in both horizontal and vertical directions (perpendicular and parallel to backstay forces). A uniformly distributed backstay force of the same magnitude and in opposite direction was applied along the length of the core wall, as illustrated in Fig. 2(b), to ensure the shear force was distributed uniformly along the core wall length ($L_w$) and to avoid stress concentrations at the bottom of the core wall.

The compressive strength of concrete was assumed to be 30 MPa and the secant modulus of elasticity was taken as 23,750 MPa. The tensile strength of concrete was assumed to be 1.8 MPa and Poisson’s ratio was assumed to be 0.15. All reinforcing bars had an actual yield strength of 400 MPa. An element size of approximately 300 mm was used. The uniformly distributed in-plane shear forces applied along the core wall on the left side and the equal and opposite forces along the foundation wall on the right side of the diaphragm were monotonically increased in a load-controlled mode. Further details of how the simulations were done are given in Mahmoodi [11].

**Discussion of Results**

**Load-Deformation Relationship of Diaphragms Subjected to Backstay Forces**

Results from the analyses indicated that the load-deformation relationship of concrete diaphragms subjected to backstay forces can be represented fairly accurately by a trilinear curve. Fig. 3 depicts the load-deformation relationship of the diaphragm with $L_{SS} \times L_{SD} = 18 \times 30$ m (Case 2) and different reinforcement amounts (0.5%, 1% and 2%). Prior to cracking, the load-deformation relationship was linear and identical for three diaphragms and the initial stiffness of the diaphragm remained constant. Once cracking took place, the slope of the load-deformation curve decreased as the diaphragm became softer due to the cracking. Since the diaphragms had relatively small aspect ratios ($L_{SS}/L_{SD}$), shear was dominant, hence diagonal (shear) cracks took place first in all analyzed diaphragms. Formation of diagonal (shear) cracks resulted in a small reduction in diaphragm stiffness. Once flexural cracking took place, the diaphragm became softer due to cracking and the diaphragm in-plane stiffness decreased noticeably. Thus, the slope of the load-deformation relationship of diaphragm reduced considerably. However, the relationship remained relatively linear after cracking. Significant reduction in diaphragm stiffness (significant change in the slope of the load-deformation diagram) occurred when the flexural cracks formed in the diaphragm.

As illustrated in Fig. 3, the level of reduction in diaphragm stiffness depends on the
diaphragm reinforcement amount. For diaphragms with 2.0%, 1.0% and 0.5% reinforcement, the reduction observed in diaphragm stiffness was approximately 85%, 90% and 95% in relation to the initial stiffness of diaphragm, respectively. The same trend was observed for the load-deformation relationships of other analyzed diaphragms. In addition, the diaphragm reinforcement amount has a significant influence on the diaphragm strength as expected. The diaphragm strength reduces considerably as the reinforcement amount reduces from 2.0% to 0.5%. Also, the reduction in diaphragm strength is reversely proportional to the aspect ratio \( \frac{L_{SS}}{L_{SB}} \) of the diaphragm. By increasing the diaphragm aspect ratio from 0.4 to 0.9, the diaphragm strength reduces by about 85% and 60% when the reinforcement ratio varies from 2.0% to 1.0% and by about 50% and 35% when the reinforcement ratio varies from 2.0% to 0.5%.

![Figure 3. Typical load-deformation relationships of diaphragms with different reinforcement amounts subjected to backstay forces](image)

**Shear and Flexural Contributions to Total Displacement**

Based on the results of this study, the shear deformation contributes between 70% and 90% of the total deformation depending on the aspect ratio of the diaphragm. Consequently, the flexural deformation contributes between 10% and 30% of the total displacement. Fig. 4 presents the load-deformation relationship of diaphragm with \( L_{SS} \times L_{SB} = 12 \times 30 \text{ m} \) (Case 4) and 1% reinforcement showing the contributions of shear and flexural deformations to the total displacement, which are about 85% and 15%, respectively. The average curvatures were integrated to determine the flexural deformation and the average shear strains were integrated to determine the shear deformation of diaphragms. The details of these calculations are given in Mahmoodi [11].

The shear and flexural deformations obtained from linear finite element (LFE) analysis (dashed lines) are also shown in this figure. Since the diaphragms are squat, the shear deformation is dominant. By decreasing the diaphragm aspect ratio from 0.9 to 0.4, the flexural deformation of diaphragm considerably reduces and the shear deformation approaches to the total displacement of diaphragm. The calculated shear and flexural displacements indicate that the contributions of shear and flexural displacements to the total displacement remain approximately constant for diaphragms with the same aspect ratio independent of the reinforcement amount. All analyzed cases exhibited a
similar trend for contributions of flexural deformation. The flexural deformation slightly decreases when diagonal (shear) cracks form in the diaphragm. Upon the formation of flexural cracks, flexural deformation increased accordingly. By increasing the applied load, diagonal (shear) cracks are extended in the diaphragm, which result in some reduction in flexural contribution.

Figure 4. Load-deformation relationship showing portions due to shear and flexure for diaphragms with $L_{SS} \times L_{SD} = 18 \times 30$ m (Case 2) and 2% reinforcement amount

Shear and Flexural Stiffness Reductions

The load-deformation relationship of diaphragms indicates that the shear and flexural stiffnesses of diaphragm significantly reduce by increasing the applied in-plane shear forces. The stiffness reduction factor at each load level was defined as the ratio of the secant stiffness to the initial uncrazed stiffness of diaphragm. In other words, the shear and flexural stiffness reduction factor was considered as the ratio of the linear shear or flexural displacement (obtained from linear finite element analysis) to the nonlinear shear or flexural displacement at each load level. Fig. 5 presents the shear, flexural and overall stiffness reduction factors versus load for the diaphragm with $L_{SS} \times L_{SD} = 12 \times 21$ m (Case 3) and 1% reinforcement. One of the important conclusions obtained from Fig. 5 is that the shear and flexural stiffness of diaphragm degrade simultaneously as the backstay force increases. Once the diagonal (shear) cracks occur, the diaphragm shear stiffness gradually reduces by about 20% while there is no reduction in flexural stiffness of diaphragms. Formation of flexural cracks due to the strong-axis bending of diaphragms results in a significant reduction in both shear and flexural stiffnesses of diaphragms. The overall stiffness reduction of diaphragms was noted to be similar to the shear stiffness reduction as all analyzed diaphragms were squat and shear-dominated and shear deformation formed the main portion of the total displacement.

Fig. 6 compares the shear stiffness reduction for diaphragms with the aspect ratio of 0.6 and different reinforcement amounts (Case 2). It reveals the influence of diaphragm reinforcement amount on shear stiffness reduction of diaphragms. For lightly reinforced diaphragms (0.5% reinforcement), the shear stiffness sharply reduced by about 95% after flexural cracking, which can be estimated by a straight line. For moderately and heavily reinforced diaphragms (1% and 2% reinforcement, respectively), there was a significant reduction in shear stiffness due to flexural
cracking followed by a smooth reduction as the applied load approaches the diaphragm strength. The diaphragm shear stiffness reduced by about 85% and 75% for the reinforcement amounts of 1% and 2%, respectively. Fig. 6 illustrates that the reduction in shear stiffness of diaphragm decreases as the amount of diaphragm reinforcement increases.

Figure 5. Shear, flexural and overall stiffness reduction factors, $\alpha$, for diaphragms with $L_{SS} \times L_{SD} = 12 \times 21$ m (Case 3) and 1% reinforcement amount

Figure 6. Shear stiffness reduction factor versus load for diaphragms with $L_{SS} \times L_{SD} = 18 \times 30$ m (Case 2) and different reinforcement amount

Conclusions

Nonlinear finite element analysis was used to investigate the effective stiffness of concrete diaphragms resisting backstay forces. The analysis results demonstrate that the initial stiffness of diaphragms remains constant before diagonal cracking takes place. Upon the formation of
diagonal (shear) cracks in the diaphragm, the shear stiffness reduces by about 20% while the flexural stiffness remains unchanged. When flexural cracking occurs in the flexural tension zone, both shear and flexural stiffnesses significantly reduce. The stiffness of diaphragm is a function of the demand and by approaching the diaphragm strength, the shear stiffness degrades to about 25%, 15% and 5% of the initial stiffness when the diaphragm reinforcement amount is 2%, 1% and 0.5%, respectively.

One of the important observations of the current study is that the shear and flexural stiffnesses of the diaphragm reduce simultaneously after flexural cracking of diaphragm occurs. This can be attributed to the “deep beam” action of the diaphragm. In addition, the analysis results indicate that the contribution of the shear deformation to the total deformation is between 70% and 90% depending on the shear span-to-shear depth ratio of diaphragm. Additional analyses will be carried out to further investigate the effective stiffness of concrete diaphragms subjected to backstay forces. The long-term goal of the current study is to develop a refined model for the effective stiffness of concrete diaphragms resisting backstay forces in high-rise buildings.

References