OPTIMAL CLIPPED LINEAR STRATEGIES FOR CONTROLLABLE DAMPING AND EXPERIMENT VALIDATION

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ABSTRACT

Over the past several decades, structural control has focused mostly on passive strategies and controllable passive strategies. Among the latter, the clipped-optimal control paradigm is one of the most commonly used. However, the optimization to design the optimal control law cannot consider the inherent passivity constraints of controllable dampers, which can result in frequent clipping of commanded forces. Therefore, an alternate optimal clipped linear control (OCLC) strategy is proposed and is demonstrated in simulation to provide performance superior to a standard clipped-LQR (CLQR) approach for single degree-of-freedom (SDOF) and other low-order structural models, especially in the cases where CLQR generates frequent clipping. A simulation study of a planned experiment with a cantilevered beam are performed to determine appropriate exciter/damper/measurement locations and optimization factors for the experiment, and to predict CLQR and OCLC performance for different objectives. Preliminary identification of the experimental system is performed.

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Optimal Clipped Linear Strategies for Controllable Damping and Experiment Validation

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ABSTRACT

Over the past several decades, structural control has focused mostly on passive strategies and controllable passive strategies. Among the latter, the clipped-optimal control paradigm is one of the most commonly used. However, the optimization to design the optimal control law cannot consider the inherent passivity constraints of controllable dampers, which can result in frequent clipping of commanded forces. Therefore, an alternate optimal clipped linear control (OCLC) strategy is proposed and is demonstrated in simulation to provide performance superior to a standard clipped-LQR (CLQR) approach for single degree-of-freedom (SDOF) and other low-order structural models, especially in the cases where CLQR generates frequent clipping. A simulation study of a planned experiment with a cantilevered beam are performed to determine appropriate exciter/damper/measurement locations and optimization factors for the experiment, and to predict CLQR and OCLC performance for different objectives. Preliminary identification of the experimental system are performed.

Background

Controllable dampers, also called smart or semiactive, inherit not only the controllable characteristics of active actuators but also the inherent energy dissipation nature of passive dampers [1]. Since the 1990s, many structures using semi-active dampers have appeared, such as the Kajima Technical Research Institute in Japan, built with active-variable-stiffness devices in 1990 [2]. In 2002, Ramallo et al. [3] proposed a smart base isolation strategy using controllable dampers that could effectively protect structures against extreme earthquakes without sacrificing performance during the more frequent moderate seismic events. In 2008, Liu et al. [4] proposed a new device using two controllable dampers and two constant springs to solve the problem that controlling stiffness and damping variables are hard to implement with conventional devices.

The clipped LQR strategy for determining the force to be applied by a controllable damper uses a primary controller designed for an ideal linear actuator to minimize a quadratic system response

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metric [5]. For a linear, time-invariant, dynamical system described by state equation \( \dot{x} = Ax + Bu \), where \( x \) is the state vector and \( u \) is the control input (force), LQR minimizes an infinite-horizon quadratic performance index of the form \( J = \int_0^\infty [x^TQx + 2x^Tu^TRu]dt \), in which \( Q \), \( N \) and \( R \) are weighting matrices. The optimal LQR control force is \( \dot{u} = -Kx \), where \( K = R^{-1}(PB + N)^T \), and \( P = P^T \) satisfies the algebraic Riccati equation \( A^TP + PA - (PB + N)R^{-1}(PB + N)^T + Q = 0 \).

Since the resulting control forces are not always dissipative — and, thus, not always achievable by the controllable damper — they must be clipped. Dyke et al. [1] formulate one clipping approach for a physical damping device; for the ideal damping device used herein, the force is clipped to zero when not dissipative. Let \( u_d = [u_{d,1} \ u_{d,2} \ldots]^T = -Kx \) be the desired forces (\( K \) is a linear gain matrix, possibly from LQR) and \( v_i \) be the velocity across the \( i^{th} \) device. Then

\[
\begin{align*}
\dot{u}_i(t) &= u_{d,i}(t) H[u_{d,i}(t)v_i(t)] = \begin{cases} 
    u_{d,i}(t), & u_{d,i}(t) v_i(t) > 0 \\
    0, & \text{otherwise}
\end{cases}
\end{align*}
\]

where \( H[\cdot] \) is the Heaviside unit step function.

**Proposed Optimal Clipped Linear Control**

The ground-relative displacement \( q(t) \) of a SDOF structure, excited by base acceleration \( \ddot{g} \), is given by equation of motion: \( \ddot{q} + 2\zeta \omega \dot{q} + \omega^2 q = -\ddot{g} - u \), where \( u(t) \) is a mass-normalized controllable damping force, \( \omega \) is the natural frequency and \( \zeta \) is the damping ratio. Then, CLQR’s desired force for this SDOF system is

\[
\dot{u}^{LQR}_d(t) = -K^{LQR}x = -k_q^{LQR} q - k_q^{LQR} \dot{q} = -\omega^2 q - 2\zeta \omega \dot{q}
\]  

(2)

The proposed optimal clipped linear control (OCLC) minimizes \( J \) for the nonlinear clipped system by choosing a control gain \( K \), which is likely different from that of the linear system’s \( K^{LQR} \). The OCLC desired force could be written, analogous to (2), as \( u_d = -k_q \dot{q} - k_q \ddot{q} \), but it is more convenient to parameterize relative to the LQR solution as: \( u_d = -\omega^2 (1 - \alpha) q - 2\zeta \omega (1 - \beta) \dot{q} \). Here, choosing nondimensional parameters \( \alpha = \beta = 0 \) provides the LQR solution, and \( \alpha = \beta = 1 \) is the structure with no controllable damping. The system is now piecewise linear and given by

\[
\begin{align*}
\ddot{q} + 2\zeta \omega \dot{q} + \omega^2 q &= -\ddot{g} - u(t) \rightarrow \begin{cases} 
    \ddot{q} + 2\zeta \omega \beta \dot{q} + \omega^2 \alpha q = -\ddot{g}, & u_d \dot{q} > 0 \\
    \ddot{q} + 2\zeta \omega \dot{q} + \omega^2 q = -\ddot{g}, & \text{otherwise}
\end{cases}
\end{align*}
\]

(3)

The optimization of \( J \) over the stiffness and damping coefficient pair \( (\alpha, \beta) \in \mathbb{R}^2 \) must be performed numerically given the nonlinear (albeit piecewise linear) nature of the system. OCLC reduces the mean square absolute acceleration by 23.2% relative to the CLQR (and 70% for mean square velocity and displacement, and has 12.5% reduction of mean square control force as well) with 1940 El Centro earthquake excitation. Notably, similar or better results with a stochastic Gaussian white noise process and 1995 Kobe earthquake records are also achieved.

**Preliminary study for the validation experiment**

A cantilevered beam, shown in Fig. 2, will be used as an experimental evaluation of OCLC and CLQR. The beam length \( L \) is 9.144 m; its weight \( W \) is 202.58 kg. The initial conditions are
assumed zero and the excitation is assumed to be a stochastic Gaussian white noise process. A reduced-order SDOF system of the cantilevered beam, modeled using a Rayleigh-Ritz approach, is studied to determine the locations of an exciter, a damper, and a performance metric measurement: the exciter is set at 0.9 $L$ and the damper and sensor are set at the tip so that the first mode dominates the reduced-order system and the damping ratio for reduced-order is matches the original. The mean square response metric is of either displacement, velocity, absolute acceleration or relative acceleration so that the performance of CLQR and OCLC can be compared with different objectives.

Then the LQR active control for the reduced-order system is employed to minimize the objective $J_{\text{beam}} = \mathbb{E}[y^2 + \rho u^2] = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} [y^2 + \rho u^2] \, dt$, which is the linear combination of mean square control force $u$ and mean square output $y$; the appropriate control weight $\rho$ for different outputs is chosen so that there is suitable reduction of response and reasonable tradeoff between control force and output for the original system, so that the reduced-order system is underdamped and stable, and that the output is respected, as well as control force is reasonable (the force levels here are on the order of a fraction of 1% of the beam weight).

Then, OCLC is applied for reducing one of several response metrics for the reduced-order system. Fig. 1 shows the surfaces of objective $J_{\text{beam}}$ as a function of $\alpha$ and $\beta$ when the response metric is displacement or absolute acceleration. When the metric is displacement, OCLC improves most relative to CLQR; when the metric is absolute acceleration, OCLC has a good reduction.

Preliminary experiments have tested the performance of the exciter and damper utilizing the same beam, but a viscous damper was employed in the preliminary experiments. A single pole digital filter (low-pass) was applied to the band limited white noise (BLWN) with a cut off frequency of 20 Hz. Fig. 3 illustrates the transfer function from the voltage to the shaker force. (The peak at about 1 Hz was intentionally added to the BLWN to examine stiction in the viscous damper, and will be removed in future testing.) Transfer function zeros around 10 Hz and 25 Hz appear because the shaker was located at (or near) an anti-node for the second and third modes (the first three modal frequencies are roughly 1.4 Hz, 9.375 Hz, and 25 Hz).
Conclusions and Future Work

An optimal clipped linear control approach has been proposed herein for determining optimal controllable damping strategies. This approach uses a quadratic performance metric similar to LQR control but optimizes the metric for the nonlinear system with the controllable damper. Then the theoretical study of the experiment for a cantilevered beam is presented and different performances of OCLC and CLQR are indicated. Future work includes: extensions to more complex structures; a validation experiment to evaluate OCLC and compare it to CLQR and then hybrid MPC to verify the theoretical results.

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