Leveraging Estimation Performance for Sensor Selection in Wireless Structural Control

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Wireless Control

**Advantages**
- Leverages distributed onboard sensing and communication
- Potentially more robust to sensor/controller failure

**Disadvantages**
- Communication Latency
- Data Loss

**Approaches**

**Decentralized Control**
Limit state feedback and include additional controllers

**Sensor Selection**
Limit sensor feedback requirements while still minimizing the error in the estimate
Wireless Control Approaches – Sensor Selection

• Limit sensor feedback requirements while still minimizing the error in the estimate
  - sensor numbers, reliability

• Traditionally challenging problem because requires considering a significant number of combinations

Kalman Estimator
Limit the state estimate error covariance in the presence of noise

\[ L = \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix}_{n \times m} \]
Sensor Selection Algorithm

Goal is to select sensors in a more systematic way

- A sparsity promoting penalty function is incorporated into the objective function to be minimized

\[
\min_{\mathbf{L}} J(\mathbf{L}) + \gamma g(\mathbf{L})
\]

Cost Function – Estimation Error Covariance

\[
J(\mathbf{L}) = \frac{1}{2} \text{tr} \left\{ \mathbf{P}_{k|k} \right\}
\]

Penalty Function

Column Sparsity

\[
g(\mathbf{L}) = \text{card}(\| \mathbf{L}_1 \|_2, \cdots, \| \mathbf{L}_m \|_2)
\]

Sparsity Parameter

\(\gamma > 0\), performance vs. sparsity tradeoff
Wireless Control Benchmark (Sun et al. SCHM, 2015)

Sensor feedback dictates latency (4 sensors, TDMA = 8ms)
- Reducing sensor feedback limits delays in the controller design

Compare original and sensor selection approach on closed-loop control performance
Sensor Selection Application

KF Sparsity as a Function of $\gamma$

Cost Function as a Function of $\gamma$
Come see my Poster!

Today Poster Session:

- **Time:** 5:15 – 7:00 pm
- **Room:** Pasadena (Exhibit Hall)
- **Poster location:** Number 132
Estimation Algorithm

Discrete-time Kalman Estimator

◦ Minimized the posteriori estimate error covariance

\[ E[x[k] - \hat{x}[k]][x[k] - \hat{x}[k]]^T \]

◦ Resulting solution is the Kalman gain, \( L \)

◦ Estimator formed:

\[
\begin{align*}
\bar{x}(k) &= \Phi \hat{x}(k-1) + \Gamma u(k-1) \\
\hat{x}(k) &= \bar{x} + L(y(k) - H \bar{x}(k) - Du(k-1))
\end{align*}
\]

\[
L = \begin{bmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix}_{n \times m}
\]
Optimal Sensor Selection

Approach:

\[
\begin{align*}
\text{minimize} & \quad J(F) + \gamma g(G) \\
\text{subject to} & \quad F - G = 0
\end{align*}
\]

As \( \gamma \) increases, the Alternating Direction Method of Multipliers (ADMM) is initialized by the optimal feedback gain at the previous \( \gamma \)

L-minimization Step

G-minimization Step

Dual variable update (Lagrangian)

Discrete-time approach simplifies the L-minimization solution

• Advantages:
  - No assumptions about sensor selection required \textit{a priori}
  - Allows for a balance of sparsity and performance
