An Integrated Discrete-Time Compensator for Real-Time Hybrid simulation

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Outline

- Motivation (servo-hydraulic delay compensation)
- Feedforward compensator
  - Design procedure
  - Performance
- Feedback compensator
  - LQG controller + Kalman estimator
- Application on Mass Damper Tests
- Summary
Motivation

Servo-hydraulic actuators experience freq-dependent variations in both amplitude and delay. Can we obtain a compensator with reliable broad-band performance?
Motivation for FF-FB controller

\[ e = r - \hat{y} \]

Feedforward (FF) Compensator

Feedback (FB) Compensator

Disturbance (uncertainties in specimen, …)

Process Noise

Measurement Noise

Plant (actuator + specimen)

\( u \)

\( u_{FB} \)

\( e = r - \hat{y} \)

32 in

25 in
Feedforward (FF) compensator

**Design Procedure**

Auto-Regressive with Exogenous (ARX) model structure is defined in discrete-time domain and can be written as below:

\[ y(t) + a_1 y(t-1) + \cdots + a_{n_a} y(t-n_a) + e(t) = b_1 u(t-n_k) + \cdots + b_{n_b} u(t-n_k-n_b+1) \]

- \( y \): output
- \( u \): input
- \( e \): disturbance (noises)
- \( n_a \): number of system poles
- \( n_b \): number of system zeros plus one
- \( n_k \): dead time in the system

Continuous-time model:

\[ y(t) = c_0 D_0 u + c_1 D_1 u + c_2 D_2 u + \cdots \]

- \( y \): output
- \( u \): input
- \( D_n \): differential operator \( d^n/dt^n \)
Feedforward (FF) compensator

Discrete-time ARX model

\[ T(z^{-1}) = \frac{b_1 + \cdots + b_{n_b} z^{-n_b + 1}}{1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a}} = \frac{B^{**}(z^{-1}) \cdot B^{-*}(z^{-1})}{A^*(z^{-1})} \]

- A pure inverse of \( T(z^{-1}) \), if stable, is a perfect compensator

\[ G_{FF}(z^{-1}) = \frac{1}{T(z^{-1})} = \frac{A^*(z^{-1})}{B^{**}(z^{-1})} \cdot \frac{1}{B^{-*}(z^{-1})} \approx \frac{A^*(z^{-1})}{B^{**}(z^{-1})} \cdot H(z^{-1}) \]

\[ H(z^{-1}) = h_1 + h_2 z^{-1} + \cdots + h_n z^{-n+1} \] (Finite Impulse Response (FIR) filter)

Continuous time model

\[ T(s) = \frac{1}{\prod_{i=1}^{m} (s - p_i)} \]

- A pure inverse of \( T(s) \)

\[ G_{FF}(s) = \frac{1}{T(s)} \]

\( T \) is proper \( \rightarrow 1/T \) is improper (cannot be realized)

Feedforward FIR compensator

Objective:
\[ \min \left( \left| \text{Frequency response} - 1 \right| \right) \]

Constraints:
- Steady-State gain = 1
- Passband magnitude = 1
- Passband phase = 0

Optimization:
- Least Square (LS)
- Weighted Least Square (WLS)
- MiniMax

Finite Impulse Response (FIR) filter:

\[ H(z^{-1}) = h_1 + h_2 z^{-1} + \cdots + h_n z^{-n+1} \]

\[ T_{com}(f_i) = \frac{\|T_{com}(f_i)\| - 1}{\|T_{com}(f_i)\|} \]

Magnitude

Phase

Tolerance (\(\varepsilon\))

Passband (\(0 < f < f_{\text{cut off}}\))

Stopband (\(f_{\text{cut off}} < f\))

Nyquist frequency (\(f_N\))
### Experimental Study

**Feedforward FIR compensator: Performance study**

<table>
<thead>
<tr>
<th>Time Delay (ms)</th>
<th>Relative RMS error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time delay ($\omega$) = $-\frac{\text{arg}[T(\omega)]}{\omega}$</td>
<td>$\text{RMSE}(%) = \frac{\sqrt{\left(\sum_i (d^m_i - d^r_i)^2\right)}}{\sqrt{\sum_i (d^r_i)^2}} \times 100$</td>
</tr>
</tbody>
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- **Graph (a):** $n = 200, f_c = 96$ Hz
- **Graph (b):**

- **Summary:**
  - All the FIR compensators are designed based on WLS optimization scheme.
Feedforward FIR compensator Performance

Input: white noise with **bandwidth** = 0~30 Hz and **max. amp.** = 0.2 in

<table>
<thead>
<tr>
<th>No compensator is applied</th>
<th>FIR compensator: MM, $n = 100$, $f_c = 32$ Hz</th>
</tr>
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<tbody>
<tr>
<td>Time delay = 21.1 ms</td>
<td>Time delay = 0.7 ms</td>
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Feedforward Feedback compensator

- Determining the optimal feedback gain ($K$)
  - Minimize Cost function: $J = \sum_i x_i^T Q x_i + u_i^T R u_i$
    - $Q$: States ($x$) weighting matrix
    - $R$: Control signal ($u$) weighting matrix
    - $\|e\|^2$

- Kalman filter performance
  - No apparent delay has been introduced
  - Reduces the noise content
NASA Disruptive Tuned Mass Damper (DTM)

Experimental substructure

Numerical substructure

\[ M \ddot{x}_{i+1} + C \dot{x}_{i+1} + R^a_{i+1} + R^f_i = F_{i+1} \]

\[ R^e_{i+1} (\text{Force}) \]

\[ x_{i+1} \]

\( x_{i+1} \) (Disp.)
Experimental Setup
RTHS of DTM

Delay (or lead) $\approx 0.3 - 0.6\text{ms}$ (a group of Eq records)
Damping Effect

- Response reduction (Kobe): 31%--Max; 45%--RMS
Summary

- RTHS is a powerful and cost effective tool to investigate the dynamic behavior of both the overall system and the components.

- An effective discrete-time compensator design is achieved by using a FF-FB framework. The discrete-time formulation:
  - Present a different technical challenge comparing to continuous-time formulation;
  - Eliminate the possible errors during time integration of compensator in real-time execution;
  - Provide a systematic way to obtain the compensator with desired performance demand (cut-off freq, delay, rms error).
Thank You!