Efficient Estimates of Uncertainties in Tsunami Inundation Forecasts

Harrison T.-S. Ko¹, Harry Yeh², Will D. Mayfield, Juan M. Restrepo, Michael Dumelle, Claudio Fuentes

¹Faculty Research Assistant, Dept. of Civil Engineering, Oregon State University
²Professor, Dept. of Civil Engineering, Oregon State University

Wednesday, June 27th
Nonlinear Shallow Water Wave Model

Output Data

Green's Function Approach

Uncertainty Estimates

Mean $\eta$

Var($\eta$)
Objectives

1. Develop low-cost alternative method for Monte Carlo Simulation in nonlinear shallow water simulations using Green’s functions

2. Estimate uncertainties from nonlinear simulation output data; estimate uncertainties independently from nonlinear model
Green’s Functions - Numerical

- Impulse response of linear numerical model
- Response is recorded from model run and stored as data matrix or Green’s functions
- Linear combinations of Green’s functions can be used to produce model output without running the model
Green’s Function Database

Numerical Green’s Functions (model impulse response) are pre-computed and stored using the Linear Perturbation Model.

Forecast

A forecast is simulated using a complex nonlinear forecast model. The output data from the simulation is stored as the reference case.

Ensemble Forecasting

The Green’s Functions are recombined to produce ensemble forecasts for various perturbed conditions almost instantaneously. Ensembles are used for variance calculations and uncertainty analysis.

Kriging

Ordinary kriging is the method employed to interpolate statistical results to the entire domain.

Application

Uncertainty estimates for coastal hazard events can inform decision makers and stakeholders to improve resilience in coastal communities.
NHWAVE Simulation

Initial condition is an ellipsoid gaussian hump of water with zero velocity

Parameters for amplitude and shape can be easily perturbed
Linearization of Problem - Part 1

Nonlinear shallow water equations:
\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}[(H + \eta)u] + \frac{\partial}{\partial y}[(H + \eta)v] = 0
\]
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial \eta}{\partial x}
\]
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial \eta}{\partial y}
\]

Expand:
\[
\eta = \underbrace{E_o}_{\text{nonlinear solution}} + \underbrace{\tilde{\eta}}_{\text{perturbation}}
\]
\[
u = \underbrace{U_o}_{\text{nonlinear solution}} + \underbrace{\tilde{u}}_{\text{perturbation}}
\]
\[
\nu = \underbrace{V_o}_{\text{nonlinear solution}} + \underbrace{\tilde{v}}_{\text{perturbation}}
\]
Linearization of Problem - Part 2

Scaling argument ($\epsilon << 1$):

\[ E_o, U_o, V_o \sim O(1) \]
\[ \tilde{\eta}, \tilde{u}, \tilde{v} \sim O(\epsilon) \]
\[ \tilde{\eta} \tilde{u}, \tilde{\eta} \tilde{v}, \tilde{u} \tilde{v} \sim O(\epsilon^2) \]

Consider only $O(\epsilon)$ terms to obtain Linear Shallow Water Perturbation Equations (LSWPE):

\[
\frac{\partial \tilde{\eta}}{\partial t} + \frac{\partial}{\partial x} (H \tilde{u} + E_o \tilde{u} + \tilde{\eta} U_o) + \frac{\partial}{\partial y} (H \tilde{v} + E_o \tilde{v} + \tilde{\eta} V_o) = 0
\]
\[
\frac{\partial \tilde{u}}{\partial t} + \frac{\partial}{\partial x} \left( U_o \tilde{u} + g \tilde{\eta} \right) + V_o \frac{\partial \tilde{u}}{\partial y} + \tilde{v} \frac{\partial U_o}{\partial y} = 0
\]
\[
\frac{\partial \tilde{v}}{\partial t} + U_o \frac{\partial \tilde{v}}{\partial x} + \tilde{u} \frac{\partial V_o}{\partial y} + \frac{\partial (V_o \tilde{v} + g \tilde{\eta})}{\partial y} = 0
\]

Solve for $\tilde{\eta}, \tilde{u}, \tilde{v}$

$E_o, U_o, V_o$ - From nonlinear model data
Repeated model runs with initial impulse at source points (center) is recorded at receiver points (right).

Impulse responses are stored as Green’s functions and later linearly combined for ensemble forecasting.
Fast Ensemble Forecasting

Linear GF Combination

Reference Solution (NHWAVE)

Perturbation Time Series (LSWPE)

Green's Functions

Source

Solution
Demonstration (at point of interest)

1000 ensemble members are computed for time series of free surface elevation at a point of interest. The different realizations (center) are used to compute a time series of variance (bottom).
Results - Efficiency (test case)

From our demonstration \((N = 128^2, G_s = 1941)\)

- NHWAVE (single model run)
  - 1 processor = \(\sim 4300\) s
  - 8 processors = \(\sim 1400\) s

- GF linear combination (1 Processor)

<table>
<thead>
<tr>
<th>#POI</th>
<th>M = 1</th>
<th>M = 10</th>
<th>M = 100</th>
<th>M = 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.28 s</td>
<td>0.93 s</td>
<td>7.7 s</td>
<td>71 s</td>
</tr>
<tr>
<td>10</td>
<td>1.2 s</td>
<td>7.5 s</td>
<td>74 s</td>
<td>730 s</td>
</tr>
<tr>
<td>100</td>
<td>9.1 s</td>
<td>90 s</td>
<td>720 s</td>
<td>7400 s</td>
</tr>
</tbody>
</table>

![Graph showing Computational Time vs. Grid Size (log-log)](image-url)
Results - Performance

Comparison of variance time series estimates using our approach to Monte Carlo simulation estimates using NHWAVE (\(M = 30\))

Coefficient of determination (\(R^2\)) is used to evaluate fit. Representative time series comparisons are shown for (a) lower, (b) middle, and (c) upper quartiles of \(R^2\) distribution.
Summary

- A concept for a methodology for using Green's functions to estimate nonlinear shallow water model uncertainties was developed.
- Our approach produces variance estimates at points of interest at a much lower time cost than traditional Monte Carlo simulation.
- Performance comparison between our approach and Monte Carlo (for our demonstration) showed a wide range of $R^2$ values: lower quartile: $R^2 = 0.296$, median: $R^2 = 0.697$, upper quartile: $R^2 = 0.913$. 