Multi-Node Gradient Inelastic Element Formulation for Structural Collapse Simulation

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1. INTRODUCTION
2. MULTI-NODE GRADIENT INELASTIC ELEMENT
3. EVALUATION EXAMPLES
4. SUMMARY AND CONCLUSIONS
INTRODUCTION

Structural Collapse

Challenge: Simulation of Structural Softening
INTRODUCTION

Modeling of Structural Members

- Beam-Column Finite Elements
  - Concentrated-Plasticity Elements
    - Elastic elements with end nonlinear springs
  - Distributed-Plasticity Elements
    - Displacement-Based Formulations
      - Prescribed displacement shape functions
    - Force-Based Formulations
      - Strictly satisfy force equilibrium equations
In the Presence of Softening Materials:

- Pathogenies of Classical Beam Theory
  - Loss of section strain filed continuity under continuous force field → strain singularity → strain localization → contradicting experimental data showing damage spread
  - Solution multiplicity
- Implications for Distributed-Plasticity Elements
  - Loss of response objectivity – solution divergence with mesh refinements
  - Solution algorithm instabilities and failures
Employs gradient nonlocality relations to associate macroscopic section strains with material section strains

Produces objective local/global responses

However, what if multiple elements are needed to represent a single member?

• To solve the nonlocality relations, the elements must be able to communicate

→ Our Solution: multi-node GI element
MULTI-NODE GRADIENT INELASTIC ELEMENT

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### MULTI-NODE GRADIENT INELASTIC ELEMENT

**Gradient Inelastic Beam Theory**

#### Force Equilibrium Equations

\[
\begin{align*}
\frac{\partial N(x)}{\partial x} &= 0, \\
\frac{\partial V(x)}{\partial x} &= 0, \\
\frac{\partial M(x)}{\partial x} + V(x) &= 0
\end{align*}
\]

#### Section Constitutive Relations

\[
\begin{align*}
\vec{D}(x) &= \vec{f}_{\text{ms}} \left( \vec{d}^e(x) \right) \\
\vec{d}^e(x) &= \begin{bmatrix} \epsilon_o(x) & \kappa_o(x) & \sigma_o(x) \end{bmatrix}^T
\end{align*}
\]

#### Nonlocality Relations

\[
\begin{align*}
\vec{d}(x) - \frac{I_c}{2} \vec{d}_{,xx}(x) H(\dot{W}_s(x)) &= \vec{d}^e(x) \\
\dot{W}_s(x) &= \dot{D}(x) \dot{d}^e(x)
\end{align*}
\]

#### Strain-Displacement Equations

\[
\begin{align*}
\epsilon_o(x) &= u_{o,x}(x) \\
\phi_o(x) &= \theta_{,x}(x) \\
\gamma_o(x) &= v_{o,x}(x) - \theta(x)
\end{align*}
\]

- Damage spreading is controlled by a characteristic length, \( l_c \)
MULTI-NODE GRADIENT INELASTIC ELEMENT
Solution of Nonlocality Relations

- In **two-node** GI element: need two boundary conditions

\[
\dot{d}(x) - \frac{l^2}{2} \dot{d}_{xx}(x) H(\dot{w}_s(x)) = \dot{d}^e(x)
\]

\[
\dot{d}(x = 0) = \dot{d}^e(x = 0)
\]

\[
\dot{d}(x = L) = \dot{d}^e(x = L)
\]

- In **multi-node** GI element: need two **new boundary conditions** per intermediate node

\[
\dot{d} = \dot{d}^e
\]

\[
? = ?
\]

\[
\dot{d} = \dot{d}^e
\]
MULTI-NODE GRADIENT INELASTIC ELEMENT
Solution of Nonlocality Relations

- New Boundary Conditions

\[
\begin{align*}
\dot{d}(x^-) &= \dot{d}(x^+)
\end{align*}
\]

\[
\begin{align*}
\dot{d}_x(x^-) &= \dot{d}_x(x^+)
\end{align*}
\]

\[
\begin{align*}
\dot{d}_{xx}(x^-) &= \dot{d}_{xx}(x^+)
\end{align*}
\]

\[
\begin{align*}
\dot{d}^e_{xx} &= \dot{d}^e_{xx} + \dot{d}^e_x(x^-) - \dot{d}^e_x(x^+)
\end{align*}
\]

Which two?!
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EVALUATION EXAMPLES
Reinforced Concrete Column

2900 kN

Model 1
BCs 1: Equal 0\textsuperscript{th} and 1\textsuperscript{st} der. ✓

Model 2
1 two-node GI
1 four-node GI

BCs 2: Equal 2\textsuperscript{nd} and 3\textsuperscript{rd} der. ✓

Model 2 - BCs 1
Model 2 - BCs 2
EVALUATION EXAMPLES
Reinforced Concrete Column with Point Loads

Model 1
Model 2

BCs 1: Equal 0\textsuperscript{th} and 1\textsuperscript{st} der. ✓
BCs 2: Equal 2\textsuperscript{nd} and 3\textsuperscript{rd} der. ✗

3 two-node GI
1 four-node GI

Discontinuity

\( d = d^e \)
EVALUATION EXAMPLES

Reinforced Concrete Water Tank Tower

Model 1
- One two-node GI
- BCs for Multi-Node GI:
  - Equal 0\textsuperscript{th} and 1\textsuperscript{st} derivatives

Model 2
- Eight two-node GI

Model 3
- One nine-node GI
EVALUATION EXAMPLES

Reinforced Concrete Frame with Wall

Model 1: 2 two-node GI ele.
Model 2: 1 three-node GI ele.

BCs for Multi-Node GI: Equal 0\textsuperscript{nd} and 1\textsuperscript{st} derivatives
A novel multi-node gradient inelastic force-based element formulation was proposed, which:

- Allows representation of a member using multiple interconnected subelements
- Uses gradient-based nonlocality relations to ensure continuity of the section strain fields in the presence of softening materials

The evaluation examples demonstrated the capability of the proposed element in:

- Producing objective (mesh-convergent) softening responses
- Generating continuous section strain distributions over member lengths
Discretization

- Equal 0th and 1st derivatives

\[
\begin{align*}
\dot{d}(x_L^{(i)}) &= \dot{d}(x_o^{(i+1)}) \\
\dot{d}_{,x}(x_L^{(i)}) &= \dot{d}_{,x}(x_o^{(i+1)})
\end{align*}
\]

\[
\frac{3\dot{d}(x_N^{(i)}) - 4\dot{d}(x_{N-1}^{(i)}) + \dot{d}(x_{N-2}^{(i)})}{2\Delta x_i} = \frac{-\dot{d}(x_3^{(i+1)}) + 4\dot{d}(x_2^{(i+1)}) - 3\dot{d}(x_1^{(i+1)})}{2\Delta x_{i+1}}
\]

\[
\rightarrow \dot{d}_{tot, all} = \left( [H_1]^{-1} [H_2] \right) \dot{d}_{tot, all}^e
\]
MULTI-NODE GRADIENT INELASTIC ELEMENT

Solution of Nonlocality Relations

- Discretization
  - Equal 2\textsuperscript{nd} and 3\textsuperscript{rd} derivatives

\[
\begin{align*}
\dot{\varepsilon}(x_L) &\quad - \frac{1}{2} \partial^2 \dot{\varepsilon}(x_L) = \dot{d}(x_L) \\
\dot{\varepsilon}(x_o) &\quad - \frac{1}{2} \partial^2 \dot{\varepsilon}(x_o) = \dot{d}(x_o) \\
\dot{\varepsilon}(x_{o+1}) &\quad - \frac{1}{2} \partial^2 \dot{\varepsilon}(x_{o+1}) = \dot{d}(x_{o+1})
\end{align*}
\]

\[\begin{align*}
\dot{\varepsilon}(x_L) &\quad - \dot{d}(x_L) = \dot{d}(x_L) - \dot{d}(x_{o+1}) = \dot{d}(x_{o+1}) - \dot{d}(x_{o+2}) \\
\dot{\varepsilon}(x_o) &\quad - \dot{d}(x_o) = \dot{d}(x_o) - \dot{d}(x_{o+1}) = \dot{d}(x_{o+1}) - \dot{d}(x_{o+2}) \\
\dot{\varepsilon}(x_{o+1}) &\quad - \dot{d}(x_{o+1}) = \dot{d}(x_{o+1}) - \dot{d}(x_{o+2})
\end{align*}\]

\[
\dot{\varepsilon}(x_N) = \left[ H_1^{-1} H_2 \right] \dot{d}_{\text{tot,all}}
\]

\[
\begin{align*}
\dot{\varepsilon}(x_{N-1}) &\quad - \dot{d}(x_{N-1}) + \dot{d}(x_{N-2}) = \dot{d}(x_{N-1}) - \dot{d}(x_{N-2}) \\
\dot{\varepsilon}(x_{N-1}) &\quad - \dot{d}(x_{N-1}) + \dot{d}(x_{N-1}) = \dot{d}(x_{N-1}) - \dot{d}(x_{N-2}) \\
\dot{\varepsilon}(x_{N-1}) &\quad - \dot{d}(x_{N-1}) + \dot{d}(x_{N-1}) = \dot{d}(x_{N-1}) - \dot{d}(x_{N-2})
\end{align*}
\]

\[
\begin{align*}
\frac{3 \dot{\varepsilon}(x_N) - 4 \dot{\varepsilon}(x_{N-1}) + \dot{\varepsilon}(x_{N-2})}{2 \Delta x} - \frac{-\dot{d}(x_{N-1}) + 4 \dot{d}(x_{N-1}) - 3 \dot{d}(x_{N-2})}{2 \Delta x_{N-1}} &= \frac{3 \dot{\varepsilon}(x_{N}) - 4 \dot{\varepsilon}(x_{N-1}) + \dot{\varepsilon}(x_{N-2})}{2 \Delta x} - \frac{-\dot{d}(x_{N-1}) + 4 \dot{d}(x_{N-1}) - 3 \dot{d}(x_{N-2})}{2 \Delta x_{N-1}}
\end{align*}
\]

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